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Review

Counting on automatization: The automatized counting theory of mental arithmetic

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ABSTRACT

Counting—enumerating sets by establishing one-to-one correspondence with an internal counter—is fundamental to the invention and development of mathematics. Yet, dominant models assume that counting is abandoned over the course of learning as children achieve proficiency with basic arithmetic calculations (e.g., 2 + 3), being replaced by the retrieval of associations between operands (e.g., 2 + 3) and answers (e.g., 5) from long-term memory. Here we challenge this assumption, arguing that counting remains at the very heart of arithmetic fluency even in adults alongside associative retrieval. Using mental addition as our test case, we present evidence that associative models fail to account for key behavioral and neuroimaging findings. We then put forward the *automatized counting hypothesis*, a novel framework proposing that counting procedures initially used by children become progressively accelerated through practice until they operate unconsciously and effortlessly. Automatized counting may become so efficient that it may generate answers to simple addition problems as fast as (or faster than) retrieval of associations from memory, particularly for problems with small operands. Recognizing the role of counting in arithmetic development explains a range of problematic data for purely associative accounts. We present behavioral and neuroimaging evidence supporting our model and discuss its theoretical, educational, and clinical implications. Overall, counting should not be seen as a steppingstone to be abandoned, but as an enduring foundation of arithmetic skills. This view challenges the assumption that automaticity necessarily relies on associative retrieval, suggesting that procedural automatization might be fundamental to skilled performance across domains of symbolic knowledge.

> "I regard the whole of arithmetic as a necessary, or at least natural, consequence of the simplest arithmetic act, that of counting" Dedekind, 1901, p.4

The invention of arithmetic has transformed human civilization by providing foundational mathematical tools upon which modern societies are built. Yet, that invention would not have been possible without the emergence of counting behavior among our species (Everett, 2019). Counting requires enumerating sets of objects by putting each in correspondence with an internal counter, whether

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non-verbal (e.g., tally marks or fingers) or verbal (e.g., number words or numerals). As such, acts of counting are largely recognized as milestones in the development of early arithmetic skills in children (Baroody, 1987).

Nonetheless, counting is typically not seen as instrumental in achieving arithmetic fluency, i.e., the ability to come up with answers to basic arithmetic operations such as 2 + 3 rapidly and accurately. On the contrary, dominant models that have shaped numerical cognition for over 50 years assume that arithmetic fluency is achieved by abandoning counting algorithms as learners become proficient (e.g., Ashcraft, 1992; Chen & Campbell, 2018; Chen & Siegler, 2000; Logan, 1988). Instead, counting is often thought to be entirely replaced by retrieval of associations between operands (e.g., 2 + 3) and answers (e.g., 5) from long-term memory. Such models are influenced by the view that human memory is associative in nature, such that items repeatedly experienced in close spatial or temporal contiguity become represented within networks of associations. Therefore, achieving arithmetic fluency would be made possible by retrieving these associated items, which would be faster than any counting procedures.

Here we argue that, contrary to what these popular models posit, counting may remain at the heart of basic arithmetic processing even in adults, alongside associative retrieval. Focusing on simple mental addition, we propose that achieving arithmetic fluency may result, at least in part, from the acceleration of a counting procedure that becomes (1) extremely fast, (2) effortless, (3) obligatory, and (4) unconscious, thereby exhibiting all characteristics of an automatic process (LeFevre et al., 1988; Zbrodoff & Logan, 1986). As we will show, positing the coexistence of such an automatization of counting in mental addition alongside associations in long-term memory allows for accounting for a range of behavioral and neuroimaging findings inconsistent with associative models alone.

In what follows, we begin by reviewing how counting emerges in children, before reviewing the most influential associative models of mental arithmetic and how they came to posit a disappearance of counting over learning. We subsequently discuss the issues these models face when attempting to account for various findings, including changes in the problem-size effect, differences in processing addition and multiplication, and neural mechanisms underlying addition during development. We then propose that many findings can be accounted for by positing that counting is not abandoned but rather becomes automatized. We term this account the *automatized counting hypothesis*. After specifying boundary conditions for such an automatized counting procedure, we discuss theoretical implications as well as implications for educational practices and for the conceptualization of arithmetic learning difficulties.

The emergence and development of counting in children

As soon as children are able to move objects around, from around 9 months of age, they begin to put some together, set others aside, and sometimes reunite all objects into a single collection (Starkey, 1981). From these basic behaviors, children learn to treat objects as distinct units and to track sets through separation and recombination. Such early experiences with manipulating and grouping objects provide the groundwork for later, more explicit acts of counting (Carpenter & Moser, 1984).

To understand the act of counting a set of objects, children must construct the cardinality principle, which is the understanding that the last number word uttered during counting corresponds to the total number of objects in the set. Before mastering this principle, children operate primarily within a logic of ordering and often interpret the last number word as referring only to the last object counted (Fuson, 1982). As Sarnecka and Wright (2013) formulated it, "it is the cardinality principle that gives number words their meanings, by making the cardinal meaning of any number word knowable from that word's ordinal position in the counting list" (p. 2). Many children do not master the cardinality principle before the age of five, and those who acquire it earlier are more likely to develop strong numerical skills over the course of development (Geary et al., 2018).

Once children become cardinal knowers, they understand that each number word corresponds to a specific quantity and can physically combine collections of objects. For example, if they have three objects on one side and four on the other, they can bring the two sets together and determine the total. Initially, children restart counting from one rather than continuing from the previously counted number. Through repeated practice, this physical manipulation lays the groundwork for developing more efficient counting strategies and the eventual ability to add quantities mentally.

Finger counting plays a pivotal role in helping children transition from physical to mental manipulation of quantities. In a recent longitudinal study, Krenger and Thevenot (2025) followed children from age 4½ to 7½ and found that virtually all children resort to finger counting at some point in their development. Children who use their fingers to count are more accurate in solving simple addition problems than those who do not, at least up to age 6½ (Dupont-Boime & Thevenot, 2018; Jordan et al., 2008; Krenger & Thevenot, 2024). After age 6½, children who outperform finger users are typically former finger users who have transitioned to mental strategies (Krenger & Thevenot, 2025).

Therefore, acts of counting are ubiquitous in the initial development of mental arithmetic. Counting procedures increase in efficiency as children become increasingly proficient with arithmetic (Baroody & Tiilikainen, 2013). However, no matter how efficient counting procedures may become, they are always considered to require significant attentional and executive resources (Ashcraft, 1992), which would prevent them from reaching the degree of automaticity required to support fluent arithmetic in older children and adults. This is why most models of arithmetic development posit that counting is progressively replaced by retrieval of associations between operands and answers from memory. We describe these models below.

Associative models for mental addition

Initial findings from Groen and Parkman

Groen and Parkman (1972) were among the first to investigate whether solving single-digit addition problems involves reconstructive processes (e.g., counting) or retrieval of stored facts. They found that by the end of Grade 1, the minimum addend best

predicted solution times for non-tie problems, with a slope of 410 ms per increment. This suggested that first graders solve addition problems using one-by-one counting, with each step taking approximately 400 ms.

When testing adults on similar problems, Parkman and Groen (1971) found that the minimum addend remained the best predictor of solution times. However, contrary to the results obtained with first graders, the sum of the problems accounted for almost as much variance in solution times as the min. Nonetheless, the slope was drastically smaller (20 ms) than for children (410 ms). The authors found it difficult to reconcile this shallow slope with a counting process. They therefore concluded that adults must retrieve answers from memory, with the residual 20 ms slope due to rare reversions to counting when retrieval fails. These seminal observations laid the groundwork for models in which retrieval from memory is central to arithmetic fluency.

Associative models: Retrieval via spreading activation and network interference

Building on Groen and Parkman's findings, subsequent models proposed various mechanisms to explain how arithmetic facts may be retrieved from memory in adults. For example, Ashcraft and colleagues (Ashcraft, 1982, 1992; Ashcraft & Battaglia, 1978; Ashcraft & Fierman, 1982) hypothesized spreading activation in associative networks. Testing university students on addition verification tasks, Ashcraft and Battaglia (1978) found that the sum squared best predicted solution times for non-tie problems. They concluded that this non-linear increase could not reflect counting but rather indicated search or activation spread within a memory network where operand nodes connect to possible answer nodes. When a problem is presented, activation spreads from operand nodes to all potential responses, with the correct answer receiving highest activation from both operands. The strength of these associations would then depend on problem frequency.

Investigating the assumed developmental transition between counting to retrieval, Ashcraft and Fierman (1982) tested children in grades 3, 4, and 6. Individual analyses revealed that half of third graders showed patterns consistent with counting, while the other half showed patterns suggesting retrieval. From Grade 4 onward, sum squared was the best predictor, similar to adults. The authors concluded that the transition from counting to retrieval occurs between Grade 3 and 4, with retrieval becoming increasingly automatic through development.

Whereas Ashcraft proposed that the problem size effect reflects memory search or spreading activation, Campbell (1987a, 1987b) argued that addition facts are accessed directly, with residual size effects due to interference between facts. As problem size increases, interference from competing associations would also increase, slowing retrieval. Small problems would be less sensitive to interference because associations are stronger due to earlier acquisition (Campbell & Graham, 1985) and more frequent practice (Ashcraft & Christy, 1995; Geary, 1996; Hamann & Ashcraft, 1986).

Siegler and collaborators (Siegler, 1987, 1988; Siegler & Shrager, 1984) also assumed direct retrieval but emphasized how errors during acquisition may create interference. Because larger problems require more counting steps, they elicit more errors during learning. These erroneous answers would become associated with problems, causing interference during later retrieval. Siegler's "overlapping waves model" (Chen & Siegler, 2000; Siegler, 1991) proposes that multiple strategies coexist during development, with less effective strategies gradually replaced by more effective ones. Retrieval is considered most effective and would always be attempted first. If it fails, reconstructive strategies could be used.

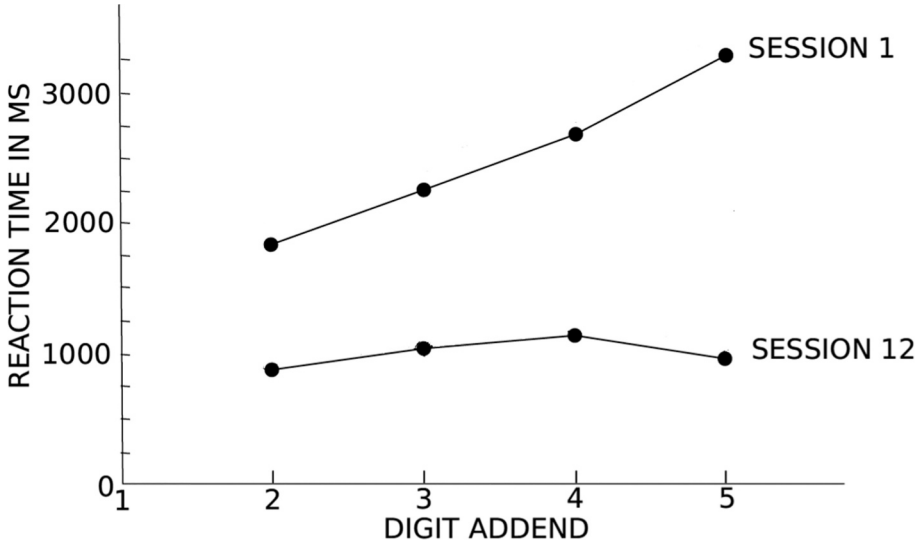

Fig. 1. Mean verification times of true problems in Logan and Klapp's alphabet numerical task, as a function of addends for Sessions 1 (top lines) and 12 (bottom lines).
Adapted from Logan & Klapp, 1991)

The instance theory of automatization

Logan (1988) proposed a distinct associative account where each problem encounter creates a new memory trace or "instance", rather than strengthening a single association. As instances accumulate, the probability of retrieving one instance during problem solving increases. Logan's model features a "horse race" between counting and retrieval: both processes launch simultaneously, and the faster wins. Initially, counting would win because no instances exist. As instances accumulate through practice, retrieval would increasingly win the race, resulting in decreased solution times and reduced size effects.

Logan notably tested these predictions using "alphabet arithmetic" tasks where participants perform mental letter-number addition (e.g., F + 4 = J) (Compton & Logan, 1991; Logan & Klapp, 1991). Initially, participants must count through the alphabet, but Logan posited that retrieval should become more frequent with practice. Logan and Klapp (1991) trained participants over 12 days to verify 40 problems. The slope associating solution times and addends decreased from 486 ms during session 1 to a non-significant 45 ms after 12 sessions (see Fig. 1). The authors therefore concluded that counting was progressively replaced by memory retrieval, which would explain the disappearance of size effects.

Summary

Classical associative models of mental addition assume a shift from costly and conscious counting procedures in childhood to retrieval-based strategies in adulthood. This interpretation is largely based on the significant decrease in problem size effects across learning and development. This is broadly consistent with self-report studies where adult participants typically report solving small addition problems by simply remembering the answer (e.g., 97% of trials in Campbell & Xue, 2001). Nonetheless, the problem size effect remains present even in expert adults. To some extent, this leftover problem size effect might be due to the sporadic use of costly non-retrieval strategies, which increases as problem size increases (LeFevre et al., 1996; Siegler, 1987). However, this idea does not explain that the problem-size effect remains present even when problems reported as solved by non-retrieval strategies are discarded. Therefore, it has been argued that residual variations in solution times may be due to retrieval-based mechanisms (e.g., Chen & Campbell, 2018), particularly interference effects (Zbrodoff & Logan, 2005). For instance, direct access to addition facts would be faster for the less interfering facts, which would be the facts involving the smallest sums. The order of acquisition could also play a role, with problems learned earlier characterized by less interference (Campbell & Graham, 1985; De Visscher & Noël, 2014). Finally, interference could depend on the number and frequency of incorrect answers associated with a problem during acquisition (Siegler & Shrager, 1984).

Issues with associative models of mental addition

Associative models have gained consensus in the field, largely because (i) they account for the landmark finding that the problem size effect decreases with learning and development (Groen & Parkman, 1972), (ii) they find support in the apparent similarity between solving small addition and small multiplication problems (Campbell & Xue, 2001), and (iii) neuroimaging studies appear to support qualitative differences in brain mechanisms between small and large problems (Sokolowski et al., 2023). In the following, we explain why there are major issues with each of these claims and why associative models fail to account for critical findings, raising the possibility that counting may still play a role in achieving arithmetic fluency.

Issues with the problem size effect

As first suggested by Groen and Parkman (1972), perhaps the most straightforward explanation of the problem size effect in young children is that it reflects a counting procedure, with increased response times resulting from step-by-step counting. If associations in a network are accessed in constant time, there is no a priori reason to assume that retrieving answers would lead to a problem size effect. Therefore, the effect should disappear as answers become retrieved from memory. This is the pattern observed for tie problems: response times increase with size at the beginning of 1st grade, but the effect quickly diminishes and is no longer observed in older children (Bagnoud et al., 2021) or adults (Uittenhove et al., 2016). This rapid commitment of tie problems to long-term memory during development has been attributed to several factors. In particular, their repeated-operand structure is thought to make them less susceptible to interference than problems involving different operands (e.g., De Visscher & Noël, 2014). Moreover, if, as suggested notably by Butterworth et al. (2001), there is a preferred order of retrieval for non-tie problems (e.g., 4 + 3 rather than 3 + 4), the comparison stage required to determine this order is absent for tie problems, which would again make them easier to represent and retrieve. A more recent explanation proposes that, early in development, tie problems are supported by symmetrical finger representations, which reduce complexity and facilitate their encoding in long-term memory (Bagnoud et al., 2025).

In contrast to tie problems, the problem size effect never entirely disappears for non-tie problems, even for small addition problems in expert adults. The small size of the effect in adults (e.g., 20 ms per increment in the smallest operand, Groen & Parkman, 1972) is often considered inconsistent with silent counting, estimated at a minimum rate of 125 ms per item (Landauer, 1962). Proponents of associative models have argued that the problem size effect for small sums in adults may stem from sporadic backup counting when retrieval fails (Groen & Parkman, 1972), a search in a table-like memory representation (Ashcraft & Battaglia, 1978), differences in the strength of associations due to variations in exposure frequency (Ashcraft & Christy, 1995; Geary, 1996; Hamann & Ashcraft, 1986), encoding of errors during development (Siegler & Shrager, 1984), or increased interference from competing arithmetic facts as problem size increases (Chen & Campbell, 2018).

Nonetheless, there are several major challenges with how associative models explain the problem size effect in adults. First, all of the explanations above predict a monotonic increase in response times as problem size increases across all problems typically considered retrieved from memory (i.e., problems with a sum ≤ 10). However, as shown in Fig. 2A adapted from Uittenhove et al. (2016), this is not the case. While response times increase from sums of 3 to 7, a plateau is observed from sums of 7 to 10. Beyond this range, response times rise steeply again as problems are increasingly solved using self-reported explicit procedures. This breakpoint around a sum of 7 roughly corresponds to the division between problems with operands ≤ 4 and those with operands > 4. When focusing only on problems for which participants report knowing the answer, Uittenhove et al. (2016) found a significant slope of 46 ms per increment in the sum for problems with operands ≤ 4. In contrast, there was no significant positive slope for problems with operands > 4 (and a sum between 7 and 10). This pattern has been replicated in several studies in both adults and children after age 10 (Bagnoud et al., 2021; Díaz-Barriga Yáñez et al., 2023; Thevenot et al., 2016). Chen and Campbell (2018) attempted to explain this plateau by suggesting that problems summing to 10 may exhibit shorter response times due to frequent drilling in school. However, even when removing these problems, a plateau remains for sums between 7 and 9 (Díaz-Barriga Yáñez et al., 2023), which cannot be fully explained by the special status of sum-to-ten problems alone (Thevenot & Barrouillet, 2020). Therefore, it remains unclear how associative models can account for the non-monotonic shape of the problem size effect for small problems in adults.

A second issue with associative models comes from the assumption that the problem size effect results from a qualitatively different mechanism in adults compared to children. While it is generally accepted that the problem size effect in young children results from an incremental counting procedure, most current associative models propose that a different mechanism explains the effect observed in adults (e.g., differences in associative strength or interference). In a recent fMRI study, Díaz-Barriga Yáñez et al. (2023) tested this assertion by examining the neural correlates of the problem size effect in 128 participants ranging from 8 years old to adulthood. That study replicated that the problem size effect decreases with age but remains significant in adults, primarily in problems with operands up to 4. Critically, however, there was a striking similarity in the brain patterns associated with the problem size effect in both young children and adults (see Fig. 2B). Increases in brain activity corresponding to increases in problem size were observed in the same regions for both adults and children. In adults, this relation was limited to problems with operands ≤ 4. Therefore, there was no evidence supporting the claim that the source of the problem size effect necessarily changes with age. If anything, that study suggests that the brain regions supporting the problem size effect in children even more strongly support that effect in adults.

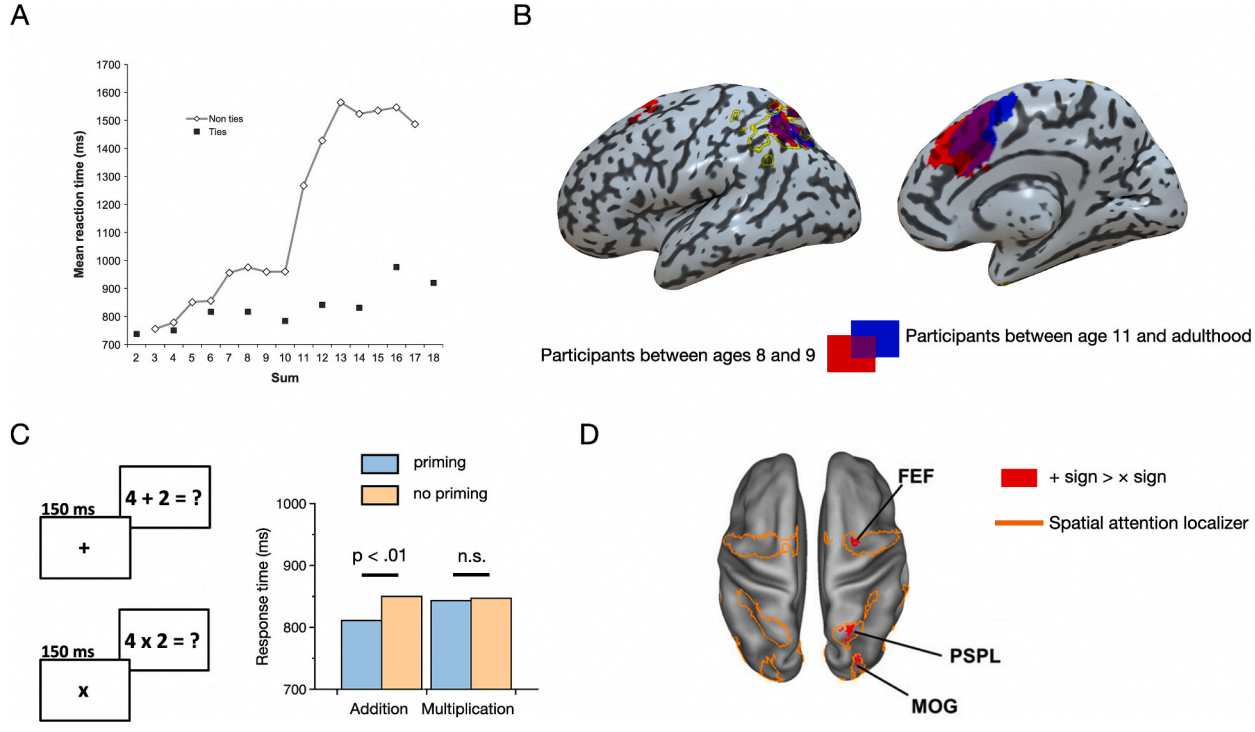

Fig. 2. Some evidence inconsistent with associative models. (A) Relation between response time and single-digit addition problem sum for tie and non-tie problems in adults (adapted from Uittenhove et al., 2016). (B) Brain regions in which activity increases with addition problem sum in 8- to 9-year-old children (considering both problems with operands ≤ 4 and operands > 4) and participants older than 11 (considering only problems with operands ≤ 4) (adapted from Díaz-Barriga Yáñez et al., 2023). (C) Illustration of the operator-priming paradigm used in Fayol and Thevenot (2012) (left) and plot of response times as a function of the priming condition for single digit addition and multiplication problems in that study (problems with operands ≤ 5) (right). (D) Brain regions associated with greater activity when adults are presented with an addition sign compared to a multiplication sign (adapted from Mathieu et al., 2018). These correspond to the Frontal Eye Fields (FEF), the Posterior Parietal Lobule (PSPL), and the Middle Occipital Gyrus (MOG).

It is important to note that observing such continuity in the neural correlates of the problem size effect does not preclude developmental changes in other brain systems involved in arithmetic processing. Indeed, longitudinal studies have documented increased hippocampal engagement during arithmetic problem solving as children transition from explicit counting to more fluent performance (Qin et al., 2014). This hippocampal involvement has been interpreted as evidence for the formation of associations in memory between problems and answers. We do not contest the fact that associations between operands and answers may be formed over development, which may explain hippocampal engagement generally (though this constitutes a reverse inference, see section on neural evidence below). However, because Qin et al. (2014) did not examine brain activity as a function of parametric increases in problem size, their findings do not speak to the mechanisms underlying the problem size effect specifically. In contrast, the similar neural signatures of the problem size effect across development (Díaz-Barriga Yáñez et al., 2023) suggest that counting may remain the primary factor driving the problem-size effect in adults as in young children.

A third issue arises when associative models explain the problem size effect by assuming it comes from overlap between problems or from differences in frequency of presentation. To test this hypothesis, Zbrodoff (1995) used an alphabet arithmetic task in which she manipulated both the overlap between problems and their presentation frequency. Zbrodoff showed that neither the overlap between problems nor the frequency of presentation created a problem size effect on its own. This is inconsistent with associative models, which predict that each factor should independently produce a problem size effect. Instead, the problem size effect only appeared when there was an interaction between frequency and degree of overlap. Therefore, it is unclear that either frequency or overlap may, on their own, produce a problem size effect.

A final issue comes from Logan and Klapp's (1991) alphabet arithmetic results, which are pivotal for the instance theory of automatization. Logan and Klapp showed a small and nonsignificant addend slope of 45 ms/addend at the end of training, which was a central argument for positing a shift from counting to memory retrieval. However, Thevenot et al. (2020) recently observed that alphabet-arithmetic tasks are characterized by the emergence of a discontinuity in the shape of the relation between addends and response times at the end of training (Chen et al., 2020; Compton & Logan, 1991; Dewi & Thevenot, 2022; Dewi et al., 2021; Logan & Klapp, 1991; Wenger, 1999; Zbrodoff, 1995, 1999), much like what is observed in standard addition tasks (Díaz-Barriga Yáñez et al., 2023; Poletti et al., 2023; Uittenhove et al., 2016). As can be seen in Logan and Klapp's results (Fig. 1), solution times tend to increase from addends 2 to 4 but decrease for addend 5 (producing the overall non-significant slope). Removing the largest addend increases the slope to 125 ms/addend, corresponding exactly to the rate of silent counting (Landauer, 1962). Thevenot et al. (2020) found similar effects, showing that response times systematically decrease with the last addend, irrespective of its size. Logan and Klapp may have incorrectly inferred a lack of problem size effect by averaging two distinct processes (counting for smallest addends and retrieval for the largest). Interestingly, the fact that retrieval may occur for the largest addend is inconsistent with the claim from associative models that retrieval should start with the smallest addend. More generally, practice with alphabet–arithmetic problems is limited in such tasks, compared to the extensive practice with real addition problems across development. It is therefore not guaranteed that automatization can emerge within such time-limited, laboratory-based interventions. These constraints further reduce the likelihood that already formed problem–solution associations are retrieved at the end of alphabet–arithmetic training. Therefore, relying on Logan's findings from alphabet–arithmetic tasks to support the plausibility of retrieval models (e.g., Fias et al., 2021; Wilkins & Rawson, 2011) may be more hazardous than is commonly assumed.

Issues with the similarity between addition and multiplication

There is little doubt that arithmetic facts explicitly learned by rote in school, such as multiplication tables, are largely retrieved from memory later on. Thus, observed similarities between solving small addition and multiplication problems in older children and adults lend apparent support to associative models. For example, expert participants typically have no conscious recollection of using a procedure when solving either $2 + 3$ or 2×3 (Campbell & Xue, 2001). Studies also show that generating the product or sum of two digits is obligatory. In other words, it happens even when the task does not require it (Jackson & Coney, 2005; J. LeFevre et al., 1991; J. A. LeFevre et al., 1988), a phenomenon predicted by spreading activation across nodes in an associative network (Ashcraft, 1987). This obligatory activation is thought to cause interference between sums and products (Winkelman & Schmidt, 1974). Expert individuals also appear to generate simple sums and products with relatively similar speed (Campbell & Xue, 2001). Finally, working memory demands do not affect the problem size effect for either operation (De Rammelaere et al., 1999, 2001; Hecht, 2002). Overall, adding and multiplying small numbers can both be seen as automatic processes.

However, the observation that small addition and multiplication problems are equally automatic does not necessarily mean they reach automaticity in the same way. This difference is unlikely to be captured by self-reports, as participants lack conscious access to automatic processes. Notably, one key distinction is that small addition problems are typically not taught by rote as tables but rather are repeatedly calculated using counting procedures in the classroom. Growing evidence suggests this distinction may influence later processing. This is evident in several behavioral signatures that appear for addition but not multiplication.

One such signature is the selective operator-priming effect, which has been replicated in several studies (Fayol & Thevenot, 2012; Mathieu, Epinat-Duclos, Léone, et al., 2018; Mathieu, Epinat-Duclos, Sigovan, et al., 2018; Poletti et al., 2021; Roussel et al., 2002). These studies have shown that presenting the arithmetic operator before the operands speeds up addition processing but has no (or reduced) effect on multiplication (see Fig. 2C). Although the effect warrants more research because its selectivity for addition (as compared to multiplication) has not always been replicated (Chen & Campbell, 2015, 2016), the weight of evidence suggests that a '+' sign alone seems to trigger a process that aids in summing, while a '×' sign does not necessarily trigger similar processing.

Another distinctive feature is the relation between addition and spatial attention, particularly toward the right side of space. Numerous studies have shown a rightward shift of attention during addition tasks (see Prado & Knops, 2024 for a review). For instance,

solving an addition problem can facilitate detecting targets presented on the right side of a screen afterward (Masson & Pesenti, 2014). This rightward attention shift has been observed during problem presentation, often after the operator (Díaz-Barriga Yáñez et al., 2020; Masson et al., 2024; Mathieu et al., 2016; Salvaggio et al., 2022) and before participants respond (Masson et al., 2024; Salvaggio et al., 2022). Critically, there is developmental evidence that such attentional shifts depend on arithmetic practice. For example, studies have found that shifts increase in strength over the course of elementary school (Díaz-Barriga Yáñez et al., 2020). Shifts are also elicited increasingly early during problem solving with learning and development, suggesting that they may play an anticipatory role (Masson et al., 2024). Finally, attentional shifts do not appear to occur with multiplication problems (Díaz-Barriga Yáñez et al., 2020; Mathieu et al., 2016). Such spatial attentional effects observed during addition problem-solving but not multiplication problem-solving remain difficult to explain for associative models.

Finally, there is neural evidence for a dissociation between small addition and multiplication. In one of the few neuroimaging studies comparing these operations directly in adults, Zhou et al. (2007) reported a striking difference in brain activity patterns despite no behavioral differences between operations. Multiplication problems were associated with increased activity in the left superior and middle temporal gyri, while addition problems corresponded to greater activity in the right parietal cortex. Direct causal evidence for operation-specific mechanisms also comes from cortical stimulation during awake brain surgery. Indeed, Semenza et al. (2017) found that no cortical site in the parietal lobe disrupted both addition and multiplication. Each operation was affected by stimulation of distinct neural sites. This demonstrates that different arithmetic operations rely on separable computational mechanisms rather than a unitary retrieval system.

Although, to our knowledge, no neuroimaging study has contrasted addition to multiplication over development, it is interesting to note that this pattern reminds of the neurodevelopmental dissociation that was observed between multiplication and subtraction (Prado et al., 2014). Indeed, that study showed age-related increase of activity in the left middle temporal gyrus for single-digit multiplication and age-related increase of activity in the right parietal cortex for single-digit subtraction. In adults, another study found a neural dissociation between operations in parietal, prefrontal, and occipital regions, leading the authors to conclude that "addition and multiplication engage different brain processes" (Rosenberg-Lee et al., 2011). This is consistent with a study showing that the relation between problem size and multivariate patterns of activity differs between multiplication and addition in adults (De Smedt et al., 2019). Finally, two other studies found evidence of a neural dissociation between the processing of a '+' sign compared to '×' sign, once again suggesting that addition and multiplication do not rely on similar cognitive mechanisms. Interestingly, presenting a '+' sign appears to elicit more activity than a '×' sign in brain regions associated with spatial attention, including the hippocampus in children (Mathieu et al., 2018) and the right fronto-parietal cortex in adults (Mathieu et al., 2018) (see Fig. 2D). Such conclusions should be interpreted with caution because they rely on reverse inferences regarding the specific role of some brain regions (which have multiple functions; Poldrack, 2011). Nonetheless, they support the idea that addition but not multiplication is linked to spatial processing, as suggested by behavioral studies discussed above.

The neural dissociation between addition and multiplication aligns with other findings where adults were trained to solve complex arithmetic problems that were identical in form but learned using different strategies (drill versus calculation, Delazer et al., 2005) or involved different operations (multiplication versus subtraction, Ischebeck et al., 2006). Although response times or error rates were often indistinguishable at the end of training, variations in learning methods or operations were associated with distinct patterns of brain activity (Zamarian et al., 2009). This illustrates that similar performance across arithmetic problems does not necessarily indicate identical processes.

Issues with neural evidence

Over the past few decades, neuroimaging research has often been seen as supporting associative models. Several studies have claimed to show that arithmetic problems assumed to be retrieved rely on brain mechanisms qualitatively distinct from those engaged by other types of problems (Sokolowski et al., 2023). For instance, the left angular gyrus is frequently considered a region that supports retrieval from long-term memory, as it tends to be more active for smaller than larger problems (Grabner et al., 2007, 2013; Stanescu-Cosson et al., 2000) or for problems where participants report knowing versus calculating the answer (Grabner et al., 2009a, 2009b; Tschentscher & Hauk, 2014). In contrast, the intraparietal sulcus is typically more active in the opposite cases (Grabner et al., 2009b; Stanescu-Cosson et al., 2000; Tschentscher & Hauk, 2014). However, there is circularity in this argument. Most studies assume a priori that certain addition problems are retrieved while others are calculated, based on problem size or self-reports (Sokolowski et al., 2023). Observing different levels of brain activity for these problems is expected due to differences in difficulty and strategies. Finding a neural difference in such cases does not, in itself, provide evidence for associative models. Rather, it simply illustrates the link between brain function and behavioral differences.

Support for associative models is often drawn from the claim that the left angular gyrus is involved in verbal retrieval, argued to indicate that some arithmetic problems are retrieved from memory (Dehaene et al., 2005). However, there are also major issues with this argument. First, none of the associative models have explicitly proposed that associations between operands and answers are verbal. Thus, it is unclear whether demonstrating a link between language-related regions and arithmetic necessarily supports these models. Second, the idea that memory retrieval is a verbal process originates from the triple-code model (Dehaene & Cohen, 1995), which states that "rote arithmetic facts belong to the general class of rote verbal memories" (p. 102). However, there is little neuropsychological evidence linking the left angular gyrus to processing small addition problems specifically. Studies suggest that lesions in language-processing areas can impair some arithmetic operations while sparing others (Dehaene & Cohen, 1997; van Harskamp et al., 2002, 2005), but evidence frequently points to a dissociation between single-digit multiplication and other operations, including single-digit addition (Smaczny et al., 2023; van Harskamp & Cipolotti, 2001). There are documented cases where lesions impair single-

digit multiplication but spare single-digit addition, and vice versa. This is inconsistent with the claim from the triple-code model that both simple addition and multiplication rely on similar rote verbal memory.

Third, the claim that both small addition and multiplication problems are solved through rote verbal memory due to greater left angular gyrus activation is problematic because it overlooks the "resource artifact" issue (Shallice, 1988). In neuroimaging, this problem complicates comparisons of brain activation across conditions. Researchers may observe activation differences and conclude that distinct cognitive mechanisms are involved. However, these differences may instead reflect varying cognitive demands rather than unique processing mechanisms. This issue arises when addition problems are categorized by problem size or self-reported strategies (Sokolowski et al., 2023). Addition problems deemed to be retrieved because they are small or self-reported as such are also systematically less resource-demanding, as shown by faster response times and higher accuracy. The same issue is present in studies comparing brain activity for arithmetic problems with differing practice levels (Bloechle et al., 2016; Delazer et al., 2003; Ischebeck et al., 2007). Any increase in activity for practiced versus less practiced problems could indicate either a change in processing mechanisms or a reduction in cognitive demands.

The resource artifact problem makes interpreting increased activity in the left angular gyrus challenging, as it may simply reflect reduced cognitive demands (Grabner et al., 2013; Prado, 2018). This interpretation aligns with the fact that the left angular gyrus is part of the default-mode network, a brain system where activity typically decreases as task demands increase (Menon, 2023). The resource artifact problem affects all studies where conditions vary according to behavioral performance, including studies where arithmetic problems classified as retrieved or calculated via self-reports were linked to different electrophysiological markers (De Smedt et al., 2009; Grabner & De Smedt, 2011; Grabner et al., 2022). For example, Fias et al. (2021) investigated changes in brain activity associated with learning alphabet arithmetic. Over 4 learning blocks, they observed a decrease in response times and an increase in the rate of problems reported as retrieved. These changes were accompanied by a progressive decrease of activity in fronto-parietal regions and an increase of activity in default-mode regions. Such a pattern was interpreted as reflecting a qualitative change in brain functioning, indicating that "with learning, the strength of the procedural network diminishes, while the strength of the retrieval network increases" (p. 31). However, changes in brain activity are confounded with changes in task difficulty. It may also be that these neural changes reflect a reduction in cognitive demands, which would also predict decreased activity in fronto-parietal regions and increased activity in default-mode regions.

Interestingly, the resource artifact problem may also make interpretation of similarity between conditions difficult. For example, Grabner et al. (2022) used EEG measures of event-related synchronization/desynchronization (ERS/ERD) to compare small addition to small multiplication problems, arguing that alpha band desynchronization may be associated with procedural knowledge and theta band synchronization may be associated with retrieval demands (Grabner & De Smedt, 2011). The problem is that this distinction comes from self-reports, which indeed show that problems that are reported to be retrieved are associated with lower alpha desynchronization and higher theta band synchronization than problems that are reported to be calculated (Grabner & De Smedt, 2011). This distinction, however, is confounded by differences in behavioral performance, such that problems that are reported to be retrieved are systematically associated with shorter response times and lower error rates than problems that are reported to be calculated. It is then unclear whether ERS/ERD distinguishes between the type of mechanisms involved or the level of cognitive effort. The results from Grabner et al. (2022) indicate no differences between small addition and small multiplication. However, because these problems do not differ in terms of behavioral performance, this similarity may be attributable to matched cognitive demands as much as shared mechanisms. In other words, the resource artifact problem obscures any neuroimaging or electrophysiological findings that rely on a difference between conditions that also differ in task difficulty, or that more generally use a measure that is confounded by task difficulty. Note that, in contrast, studies showing neural dissociations between operations despite matched behavioral performance (e.g., Rosenberg-Lee et al., 2011; Zhou et al., 2007) provide more compelling evidence for distinct mechanisms, as they demonstrate that neural differences cannot be attributed to differences in task difficulty. For example, one interesting methodological approach to address the resource artifact problem would be to compare conditions that are matched for behavioral performance while differing in strategy (Polspoel et al., 2017), though such matching requires demonstrating that the compared conditions do not differ statistically in accuracy or reaction time.

Finally, several neuroimaging studies have identified a relation between early stages of arithmetic learning and hippocampal activity in children and adolescents (Cho et al., 2011, 2012; Qin et al., 2014; Supekar et al., 2013). For example, hippocampal volume and connectivity predict improvements following arithmetic learning (Supekar et al., 2013). Because of the role of the hippocampus in declarative memory, these studies often interpret its involvement as supporting associative models. However, this type of reverse inference only provides weak evidence, as there is no one-to-one mapping between brain activity and function (Poldrack, 2011). For example, substantial evidence indicates that the hippocampus also supports spatial cognition (Buffalo, 2015), a foundational element of procedural knowledge in arithmetic development. Supporting this idea, a study observed differential activity associated with processing an addition sign compared to a multiplication sign in a hippocampal area also involved in overt spatial attention in children (Mathieu et al., 2018). This finding is challenging to explain through associative models and instead suggests that spatial procedures evoked by the addition sign may drive hippocampal activity during arithmetic learning.

Summary

We have reviewed evidence that challenges the evidence base of associative models. First, the persistence and non-monotonic shape of the problem size effect in adults challenges the idea that it results from decreased associative strength or increased interference (Barrouillet & Thevenot, 2013). Evidence suggests that the source of this effect is not qualitatively different between adults and children (Díaz-Barriga Yáñez et al., 2023). Second, several studies indicate processing differences between addition and multiplication,

including behavioral effects specific to addition (e.g., operator priming, associations with spatial processing) (Fayol & Thevenot, 2012; Mathieu et al., 2016) and neural distinctions between these operations (Rosenberg-Lee et al., 2011; van Harskamp & Cipolotti, 2001; Zhou et al., 2007). This calls into question the claim that both are systematically retrieved from memory in adults. Finally, while neuroimaging data have often been interpreted as supporting associative models, such interpretations frequently overlook confounding factors, such as resource artifacts (Prado, 2018) and the versatile role of the hippocampus (Mathieu et al., 2018). Collectively, these issues suggest that existing associative models lack the explanatory power to fully account for the diverse processes underlying small arithmetic problem solving. In the following section, we present the automatized counting model of mental addition, which, when considered along associative models, is able to explain findings that cannot be accounted for by associative models alone.

The automatized counting hypothesis

A main tenet of associative models is that the counting procedures young children use to solve small addition problems must be abandoned by late childhood and adulthood. This is because the speed and accuracy with which these problems are processed in adolescents and adults is typically considered incompatible with algorithmic procedures, and instead viewed as compatible only with direct retrieval from memory. The idea of a developmental shift from procedural to associative knowledge is not unique to mental arithmetic, it is apparent in other domains where children repeatedly face similar problems, such as reading and spelling (Davis & Evans, 2021; Rittle-Johnson & Siegler, 1999). However, while it is certainly possible that repeated use of the same procedure may lead to associations in memory (e.g., through co-activation of items in working memory, Siegler & Shrager, 1984, Shrager & Siegler, 1998), it is equally plausible that such repetition would also reinforce the very procedure being repeated, much like sensory-motor skills in typing or learning a musical instrument may become automatized with extensive practice (Logan & Crump, 2011; Shaffer, 1982). In other words, procedural automatization and associative bonds may both result from repeated practice, rather than practice leading solely to associations in memory. Below, we outline what procedural automatization might look like for mental addition, what type of automatized procedure may underlie addition problem solving in adults, and the boundary conditions of an automatized procedure for mental addition.

Plausibility of a developmental increase in procedural efficiency up to automatization

As described earlier, the development of addition skills in children typically follows a consistent progression through increasingly sophisticated counting strategies (Baroody & Tiilikainen, 2013). Several decades ago, Baroody suggested that this progression in procedural efficiency may not necessarily end with the strategies documented in developmental research (Baroody, 1983, 1984, 1994). Drawing on Anderson's ACT model (Anderson, 1982), which posits that procedures can become compiled with practice, Baroody argued that arithmetic fluency in adults may involve the automatization of schemas, rules, and principles (e.g., commutativity, $n + 0 = n$ rule) that eventually become unconscious in their compiled form (Baroody, 1994).

Although Baroody's proposal did not specifically extend to counting itself, recent research has proposed that a counting procedure might become so efficient that it reaches automaticity (Barrouillet & Thevenot, 2013; Uittenhove et al., 2016). We believe this proposal is plausible for several reasons. First, explicit counting procedures are practiced extensively by children in early elementary school. First- and second-graders might practice small addition problems around 20 to 30 times daily through class exercises, homework, and informal activities. With approximately 180 school days per year, this amounts to roughly 7,000 to 8,000 instances of counting-based addition over two years. While proponents of associative models may argue that such extensive repetition facilitates memory associations, it also provides ample opportunity for procedural automatization.

Note that the strategies favored by children when solving small addition problems (i.e., with operands that are representable on fingers) often correspond to count-all strategies consisting in counting from one until the two operands have been enumerated. For example, when solving $3 + 2$, children may represent 3 on one hand and 2 on the other and then count all raised fingers, or they may count sequentially from one to five on their fingers. For such simple problems, count-on strategies, consisting in starting the count from one of the two operands (for example for $3 + 2$ counting 4 and then 5 starting from 3) are virtually never observed in children. These strategies emerge only when children are faced with problems for which the operands can no longer both be represented on fingers, and, at that time, the strategies for very small problems are already internalized (Krenger et al, submitted).

Therefore, if one type of counting strategy has the potential to be internalized and eventually automatized by expert solvers, it is likely a count-all strategy. Indeed, there is continuity between overt and covert strategies. For example, the problem-size effect among children who had just transitioned from finger counting to mental strategies shows similar slopes before and after mentalization, despite faster response times overall (Bagnoud et al., 2025). The automatized use of internalized count-all strategies may explain why the problem size effect for small addition problems never fully disappears with expertise, and may also explain why the addition sum remains the best predictor of solution times in adulthood (Barrouillet & Thevenot, 2013; Groen & Parkman, 1972).

Another point to consider is that automatizing such a counting procedure may take time. Indeed, while counting becomes increasingly efficient during development, each counting step taking less and less time from childhood to adulthood, automatized counting may emerge only around the age of 12 or 13. This is suggested by converging results showing that addition-sign priming effects (Mathieu, Epinat-Duclos, Léone, et al., 2018; Poletti et al., 2021) and spatial biases in addition are typically not observed before that age. At the age of 12 or 13, slopes for small addition problems range from approximately 75 ms (Diaz-Barriga Yanez et al., 2021) to 98 ms (Poletti et al., 2021). Although these values remain higher than the 20–40 ms typically reported in adults (Groen & Parkman, 1972; Barrouillet & Thevenot, 2013; Uittenhove et al., 2016), this difference is likely attributable to substantial inter-individual variability among children. For example, while Poletti et al. (2021) found an average slope of 98 ms in children, one child

exhibited a slope of 637 ms. Such variability might reflect the fact that some children may have already automatized the procedure while for others automatization has not yet occurred.

As already mentioned, the shallow slope of the problem size effect in adults had originally led to rejection of the idea that explicit counting might be systematically involved, as it appears incompatible with the rate of silent verbal counting (Groen & Parkman, 1972). This suggests that any automatized counting procedure used by older children and adults would have to have become pre-verbal (Gallistel & Gelman, 1992). A potential cognitive architecture for such a procedure is detailed in the next section.

Cognitive architecture of automatized counting

Uittenhove et al. (2016) proposed a cognitive architecture for automatized counting based on elements of the Soar cognitive architecture developed by Laird et al. (1987). This automatized counting procedure has three prerequisites. First, an ordered sequence of numbers must be present in long-term memory. Second, an additive task must be initiated by a cue (e.g., an addition sign). Third, the first operand of the addition problem (e.g., an Arabic digit) must have been transcoded into an analogical representation (e.g., a set of tokens) in working memory. Importantly, there is extensive evidence that these prerequisites are likely met when single-digit addition problems are presented to adults and older children, who (1) acquire the number sequence early in math development (Fuson, 1988), (2) learn the principles and symbols of addition in early elementary school, and (3) may automatically transcode Arabic digits into analogical representations by around age 9 or 10 (Girelli et al., 2000; Rubinsten et al., 2002).

When all prerequisites are fulfilled, the automatized counting procedure is triggered. This procedure resembles a rule that matches tokens in working memory to positions on the number sequence in long-term memory. Working memory is then updated as each token is tagged with the corresponding numerical value from the number sequence. The same process is applied successively to all tokens in working memory, with the number sequence incremented for each additional token (Uittenhove et al., 2016, refer to this as a 'next-token-next-value' rule). Once all tokens from the first operand are tagged, working memory is emptied and replaced with the analogical representation of the second operand. Tokens from the second operand are then counted similarly, starting from the last position reached after counting the first operand. The process concludes when no tokens remain to be tagged in working memory, with the number corresponding to the final token representing the answer (see Fig. 3).

The procedure described above is similar to what is called a “production” in the Soar architecture (Laird et al., 1987). In Soar, productions specify the actions to be taken based on the task goal and the contents of working memory. Specifically, a production pairs certain content in working memory (e.g., a token) with an action (e.g., the 'next-token-next-value' rule), which then brings relevant knowledge from long-term memory (such as the position in the number sequence) into working memory (i.e., the numerical value). Productions are fast, unconscious, and spontaneous, and they are repeatedly activated in what is called a “decision cycle”. Decision cycles are the smallest intentional unit of cognitive activity in the Soar model. During each cycle, Soar gathers relevant information through repeated activation of production rules, ultimately leading to a decision. Because each production within the decision cycle is spontaneous and unconscious, the cycle itself operates in a similarly spontaneous and unconscious manner, continuing until it reaches a stable state (referred to as 'quiescence') and produces a response (the sum). Therefore, only the final output of the decision cycle is

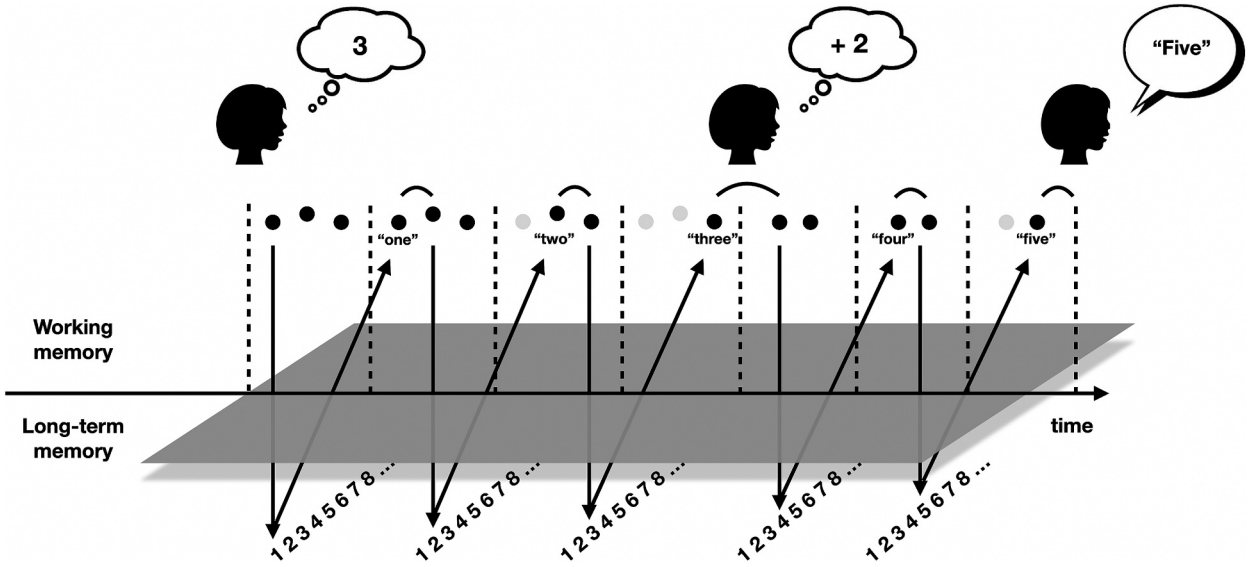

Fig. 3. Cognitive architecture of the automatized counting procedure (adapted from Uittenhove et al., 2016, and Díaz-Barriga Yáñez et al., 2023). The figure illustrates how the procedure would add the second operand ‘2′ to the first operand ‘3′ to produce an answer. Solid lines represent next-token-next-value rules. Vertical dashed lines represent production cycles. Note that the number sequence is depicted as a linear array for illustrative purposes, though the model does not require that this sequence has intrinsic spatial properties. The possibility that this sequence may be organized spatially (akin to a mental number line) is nonetheless consistent with evidence linking addition to spatial processing (see main text).

accessible to consciousness, while the production cycles that led to this output remain unconscious.

Critically, the Soar architecture has proven effective in explaining performance on tasks that involve scanning a sequence of items, such as the classic Sternberg short-term memory task (Sternberg, 1966). In this task, participants memorize a short list of items, then determine whether a probe item was part of the original list. The main finding is that response time increases linearly with the number of items in the memorized list, with each additional item adding about 40 ms to the response. This supports the concept of an exhaustive serial search. Soar models performance by implementing a succession of production rules that automatically compare each item in memory to the probe (Newell, 1992). This comparison process continues until the decision cycle completes, yielding a final response. Soar explains the linear increase in response time as resulting from the repeated application of the production rule, with each item comparison adding approximately 40 ms. Since the decision cycle runs in full before a response is generated, reaction times remain constant regardless of the position of the probe in the list. Because this entire process occurs within an automatic decision cycle, participants are unaware of the individual steps involved. Thus, the Soar architecture is well-suited to account for performance in tasks that can be solved through serial search in memory, precisely the approach proposed by the automatized counting model for mental addition.

It is important to note that, while our model requires an ordered sequence of numbers, it does not strictly require that this sequence be organized spatially (i.e., as a "mental number line"). The automatized counting procedure would function as long as the sequence is ordinal, allowing successive incrementing through production cycles. However, substantial evidence suggests that numerical magnitude is represented spatially in adults and children (Hubbard et al., 2005), and as reviewed earlier, addition is uniquely associated with spatial processing (Prado & Knops, 2024). Therefore, while not strictly necessary for our model, it is plausible that the number sequence accessed during automatized counting may have spatial properties. This interpretation is consistent with developmental evidence suggesting that children show a strengthening of associations between addition and spatial processing with age (Díaz-Barriga Yáñez et al., 2020), which may be supported by hippocampal spatial mechanisms at the beginning of learning (Mathieu et al., 2018) and a fronto-parietal attention network later in development (Mathieu et al., 2018). Recent single-neuron recordings provide additional evidence for ordered numerical representations at the neural level. Kutter et al. (2018) identified neurons in the medial temporal lobe that encode symbolic numerals with distance-sensitive tuning, where neurons tuned to a particular numeral respond progressively less to numerically distant values. This provides a potential neural substrate for the ordinal number sequence required by automatized counting. The medial temporal lobe could thus represent ordered symbolic sequences that are accessed by parietal regions during automatized counting procedures.

Why an automatized counting procedure would be advantageous compared to retrieval

Even though an automatized counting procedure may be plausible, a fundamental question is why a cognitive system would prefer an algorithm (even if that algorithm is automatic) when retrieval of an instance is possible. We believe there are several possible answers.

First, using a fast counting procedure is virtually error-free because each step follows a fixed and systematic sequence. Already at the age of 6½, 80% of children correctly solve at least 80% of problems with operands up to 5, and almost all children at age 7 solve all the problems correctly (Krenger et al., submitted). Krenger and Thevenot (2025) further showed that these successful children either still rely on counting strategies at these ages or have already internalized counting (see also Bagnoud et al., 2025). By contrast, retrieving answers from a memory network might be less reliable because it is influenced by competing associations and past errors (Siegler & Shrager, 1984). Not only are fast counting procedures less error prone than retrieval, but their completion can be quicker than retrieval. Each automatized counting step takes between 20 and 45 ms, akin to rapid scanning within a memorized list (Sternberg, 1966). Assuming all other cognitive processes are equivalent, the calculation part of problem solving using automatized counting would be completed in no more than 180 ms (with 4 as the maximum operand size). This would make automatized counting highly competitive with retrieval in terms of speed while maintaining step-by-step accuracy.

Second, procedural skills offer flexibility and transferability. As emphasized by Baroody (Baroody, 1994), rules and procedures allow for greater adaptability, enabling solutions to a wider range of problems than retrieval. Automatized counting procedures can be used whenever multi-digit addition problems can be solved by decomposing the problem into smaller, manageable units. For example, $23 + 34$ could be efficiently solved by first computing $2 + 3$ for the tens and $3 + 4$ for the units, quickly arriving at 57. An approach where complex addition problems are solved by breaking them down into smaller steps seems both practical and efficient.

Third, repetitively practicing elementary procedural steps may be essential when retrieval is unreliable or fails entirely. In such cases, the absence of a fast and efficient procedural backup system could leave individuals dependent on slow, effortful, and conscious reconstructive strategies. These demanding strategies would necessarily delay responses and decisions, potentially hindering performance in time-sensitive contexts.

Finally, relying on procedural processes rather than semantic memory is well known to confer greater resilience to aging and brain damage (e.g., Cohen & Squire, 1980). Therefore, when possible, privileging a system that is preserved across the lifespan and less vulnerable to neurological insult is highly functional and adaptative. For example, it has been shown that the rate of scanning of a memorized list of items, akin to the scanning of a numerical sequence during automatized counting, is completely spared in alcoholic and schizophrenic patients (Sternberg, 1975). This demonstrates that sequential procedural operations can remain intact despite substantial impairments in higher-order cognitive or semantic systems, reinforcing the view that automatized counting constitutes a robust and adaptive foundation for basic numerical skills.

Therefore, the procedural machinery described in the automatized counting theory may act as a crucial safety net, enabling adaptive and timely responses across tasks and situations where addition operations are required. An automatized counting procedure

may not only rival retrieval in speed and accuracy but also provide a robust and adaptable framework for addition problem-solving by enhancing arithmetic proficiency, supporting learning transfer to more complex tasks, and offering a trustworthy fallback when retrieval fails.

Boundary conditions of automatized counting and what they predict

We identify at least three major boundary conditions for an automatized counting procedure, which are essential for understanding several findings in the literature. First, this procedure (centered on incrementing a sequence of numbers in memory) must be specific to the operation of addition because any potential automatization would require extensive repetition of consistent procedure. For subtraction, children typically use varied and inconsistent procedures (e.g., counting backward from the largest operand, counting up from the smallest operand, decomposing), making procedural automatization much less likely than for addition. For multiplication, while children may initially learn facts through repeated addition (e.g., 3×4 as $3 + 3 + 3 + 3$), instruction almost systematically shifts to rote memorization of multiplication tables as repeated addition quickly becomes inefficient for large facts. Therefore, neither subtraction nor multiplication is likely to trigger the automatized counting procedure that is proposed here, which is also consistent with behavioral and neural evidence showing processing differences between addition and other operations.

Second, because the automatized counting procedure operates on an analogical representation of quantity in working memory, it is constrained by the maximum size of the set that can be represented at once. Substantial evidence suggests that the span of working memory is limited to approximately four "chunks" of information in young adults (Cowan, 2001, 2010). Four is also thought to be the upper limit for the number of objects that can be detected without enumeration, through a process known as subitizing (Mandler & Shebo, 1982). Any analogical representation of a quantity exceeding four tokens may fall outside a single focus of attention and be too imprecise for the automatized counting procedure to function efficiently. Therefore, a key prediction of the automatized counting model is that the procedure would operate most efficiently for problems with operands of four or fewer. Note, however, that this boundary should be understood as a probabilistic transition rather than an absolute threshold, with the relative efficiency of automatized counting versus retrieval potentially varying across individuals based on factors such as counting practice, instructional history, processing speed, or working memory capacity.

Third, as detailed earlier, each implementation of the production rule is believed to be very fast, lasting no more than tens of milliseconds (around 40 ms per production cycle in the Sternberg task). However, the total duration of the decision cycle required to produce the answer depends on the number of production cycles: the more tokens to count, the more production cycles, and the longer the decision cycle. Thus, the model implies that for problems solved through automatized counting, the time needed to reach a sum should increase linearly with the number of tokens counted. In other words, the automatized counting model directly predicts a problem size effect, with a slope in the range of tens of milliseconds at most.

Coexistence and competition between automatized counting and retrieval of associations

The boundary conditions outlined above clarify that we do not view automatized counting as an all-encompassing model for mental arithmetic in adults and older children. Rather, we fully acknowledge that associations between operands and answers may be formed during learning and development, even for small problems. These are most certainly created through explicit instruction and rote memorization for problems that are frequently drilled in school, but they may also be formed after repeated co-activation of operands and answers in working memory, as specified in computational models (e.g., Shrager & Siegler, 1998). Our claim is that the retrieval of these associations is not the only way through which adults and older children may solve simple addition problems. Rather, we argue that automatized counting may coexist with these associations and, in some cases (i.e., for problems with operands ≤ 4), be more efficient than memory retrieval, which would explain a range of findings described earlier.

We believe that there are at least two possible architectures for a model of mental addition that assumes the coexistence of automatized counting and retrieval of associations. On the one hand, a serial architecture would involve one process being triggered only if the other fails to provide an appropriate response. However, such an architecture presents some conceptual challenges, including determining which process initiates first and explaining how failure is detected without assuming a third monitoring system (which is problematic given that both automatized counting and retrieval are believed to be spontaneous and extremely fast). On the other hand, a parallel architecture would involve both automatized counting and retrieval of associations being triggered simultaneously upon problem presentation, with the fastest process explaining behavior. This architecture offers a number of advantages. For instance, there would be no need to assume that one process precedes the other (both would operate in parallel) and no need to assume an early monitoring system to detect failure (i.e., the process that controls behavior is simply the fastest). This architecture would also allow for the possibility that automatized counting and retrieval may sometimes produce different results, potentially creating a conflict that could trigger more deliberate strategies.

This parallel framework would provide a straightforward account of the non-monotonic problem size effect observed in adults. For problems with operands ≤ 4, performance may primarily reflect automatized counting because this would be the fastest process. For problems with operands > 4 but sum ≤ 10, performance may shift to primarily reflect retrieval of associations because automatized counting may no longer be more efficient. Moreover, the relative efficiency of each pathway may also vary across individuals. For example, we believe that the available evidence suggests that automatized counting may be generally favored over retrieval for problems with operands of four or less. However, this boundary is likely graded rather than absolute, and may depend upon individual factors such as counting practice, instructional history, processing speed, or working memory capacity. Future research should examine how these factors modulate the relative efficiency of automatized counting versus retrieval across individuals and problem

types. Note, however, that both the specific nature of the interaction between automatized counting and retrieval of associations (i.e., serial or parallel) and the factors determining individual differences in pathway efficiency also remain open questions.

A final important implication of this potential parallel framework concerns the interpretation of neuroimaging findings. If both automatized counting and retrieval of associations are triggered simultaneously upon problem presentation, then neural activity patterns may reflect the engagement of both pathways even when one process ultimately controls behavior. For addition problems with small operands, this means that brain regions associated with both procedural computation (e.g., intraparietal sulcus) and memory retrieval (e.g., hippocampus) may show activation, even if automatized counting is the faster route to the answer. This contrasts with multiplication, where procedural routes play a minimal role due to rote learning of multiplication tables. Therefore, the neural dissociations between addition and multiplication reviewed earlier may reflect fundamental differences in whether procedural systems are engaged at all, rather than differences in which system ultimately produces the answer.

Explanatory power

We believe that positing the existence of the type of automatized counting described above in adults may account for a large number of findings that are challenging for associative models alone. First, it may explain the non-monotonic shape of the problem size effect for problems with sums smaller than 10. Essentially, two different mechanisms may be at play: automatized counting for problems with operands ≤ 4 and retrieval of associations for problems with operands > 4. For problems with operands ≤ 4, the problem size effect may arise from a counting procedure that is qualitatively similar to that used by younger children, but quantitatively more efficient. This is a unique prediction that aligns with findings from the largest neuroimaging study of mental addition across development (Díaz-Barriga Yáñez et al., 2023). Specifically, for problems with operands ≤ 4, developmental changes in the neural mechanisms underlying the problem size effect were primarily quantitative rather than qualitative. The same set of brain regions supported the effect from age 9 into adulthood, with these regions showing greater sensitivity to problem size in adults than in children (despite the effect being smaller behaviorally). It is also interesting to note that the problem size effect became increasingly associated with the basal ganglia, a region that has long been associated with procedural learning. Although such an argument needs to be interpreted with caution because it relies on a reverse inference, such a finding supports the idea that procedural memory may play a role in the decrease of the problem-size effect with age. Overall, these findings are consistent with the idea that solving problems with operands ≤ 4 is increasingly supported by automatized counting.

Second, positing automatized counting explains the range of findings suggesting that, despite their similar degree of fluency in adults, small addition and small multiplication are processed differently. For example, the operator-priming effect (Fayol & Thevenot, 2012) may result from the priming of procedural knowledge associated with addition but not multiplication. The relatively late emergence of this effect in development (measurable only from adolescence, Poletti et al., 2021) supports this interpretation, as procedural automatization would require extensive practice. The automatized counting procedure relies on a sequence of numbers stored in long-term memory. Substantial evidence suggests that this number sequence may be spatially oriented, with a left-to-right organization that may be evolutionary ancient (Giurfa et al., 2022). Because this spatial orientation makes addition similar to a rightward mental movement along the sequence (Hubbard et al., 2005), it is consistent with studies showing that processing addition problems is often associated with rightward shifts of attention (Prado & Knops, 2024). In contrast, behavioral and neuroimaging studies (Díaz-Barriga Yáñez et al., 2020; ; (Mathieu et al., 2018); Mathieu et al., 2016) have shown that these rightward shifts do not occur with multiplication problems, suggesting a unique association between addition and spatial processing.

Finally, we believe that the automatized counting hypothesis, being specific to addition, accounts for a number of neuroimaging findings. It is consistent with studies showing different patterns of brain activity for addition and multiplication (Mathieu et al., 2018; Rosenberg-Lee et al., 2011; Zhou et al., 2007). It is also consistent with findings that the strategies used to learn arithmetic influence the brain mechanisms that support these operations later on (Zamarian et al., 2009). Developmental changes in the problem size effect for small addition likely reflect a quantitative increase in procedural efficiency rather than a qualitative change in mechanism (Díaz-Barriga Yáñez et al., 2023). More generally, a role of procedural automatization in achieving fluency with addition might also explain why neuroimaging studies consistently show increased activity in the intraparietal sulcus (IPS) and surrounding areas with development. It has long been argued that the IPS is involved in representing numerical quantities. As such, it is believed to be involved in number manipulation and in the explicit counting strategies that children use while learning addition. Developmental neuroimaging studies consistently report increases in IPS activity over time, which would be puzzling if problems were increasingly retrieved from memory. This increase in activity has often been described as an increase in the "functional specialization" of the IPS for arithmetic (Prado et al., 2014; Rivera et al., 2005; Rosenberg-Lee et al., 2011; Schel & Klingberg, 2016). However, what this specialization exactly entails has remained unclear. We propose that this increased parietal activity reflects, at least in part, greater efficiency in procedural knowledge, including the automatized counting procedure discussed here. Once again, such a line of argumentation should be treated with caution in future studies given that it involves some degree of reverse inference. Nonetheless, we argue that positing the development of automatized counting alongside retrieval of associations better accounts for general neural changes observed during development than retrieval of associations alone.

Implications

We have reviewed evidence pointing towards the existence of an automatized counting procedure that may be as efficient as memory retrieval for solving small addition problems. We believe that this perspective has implications at the theoretical, educational and clinical levels.

Theoretical implications

The evidence we review shows that procedural automatization may occur in mental arithmetic, much like it occurs for a variety of sensory-motor skills such as driving, typing on a keyboard, or learning how to play piano (Suzuki, 2022). This raises the intriguing possibility that procedural automatization may also occur for other aspects of symbolic knowledge that become automatic with extensive practice. One example is reading. Indeed, dominant models of visual word processing (Coltheart et al., 2001) posit that word recognition in skilled readers no longer heavily relies on reconstructive phonological processing, but rather operates through a direct orthographic pathway enabling whole-word recognition. However, there is evidence indicating that sublexical letter-sound procedures may remain important, particularly in skilled readers (Booth et al., 1999; Ferrand & Grainger, 1992; Perfetti et al., 1988), suggesting that phonological information processing contributes to visual word recognition, potentially through a reconstructive procedure where orthographic input is rapidly and automatically converted to phonological code (Booth et al., 1999). Similar principles may apply to other symbolic domains where sequential procedures are practiced extensively, such as spelling (where phoneme-to-grapheme conversion rules may become automatized) or musical sight-reading (where note-reading procedures may become automatic with practice). However, the specific conditions under which procedural automatization occurs (versus being replaced by retrieval of associations) remain to be specifically explored for these domains. More generally, our proposal suggests that procedural automatization may be a fundamental mechanism underlying the development of fluency across domains of symbolic knowledge.

Finally, our framework highlights the need for future research to examine the role of subcortical structures, particularly the basal ganglia, in arithmetic automatization. Recent evidence suggests the basal ganglia may play a crucial role in the procedural execution and "chaining of operations" required for multi-step arithmetic (Saban et al., 2024; Flaumenhaft et al., 2025), and age-related increases in caudate activity have been observed specifically for small addition problems during development (Díaz-Barriga Yáñez et al., 2023). The framework proposed here predicts that the basal ganglia may be important for the automatization of counting procedures, facilitating the rapid sequential execution of counting steps. Future developmental neuroimaging studies should systematically examine subcortical as well as cortical contributions to arithmetic strategy development.

Educational implications

The notion that automatized counting may play a role in achieving arithmetic fluency has implications for education. According to retrieval models, repetitive practice of addition using reconstructive strategies should lead to the construction of retrievable knowledge associating operands to answers. This is why Fuchs et al. (2010) proposed bypassing procedural practice for children with arithmetic difficulties, recommending that they directly memorize addition facts in a manner similar to learning multiplication tables. The authors compared a training condition focused solely on rote learning with another condition that included supplementary training in counting procedures (e.g., using number lines as well as finger or verbal counting). The results showed that children who received training involving both rote and procedural learning showed greater improvements in arithmetic skills than those who underwent rote learning alone. Moreover, reducing the amount of rote learning did not reduce the beneficial effects of the training. The authors concluded that training counting strategies is a powerful tool to build both accuracy and fluency. We argue that this is not because training counting helps children construct associative representations of arithmetic facts in long-term memory. Rather, this is because it enhances the speed and accuracy of counting strategies themselves, which ultimately may lead to automatization.

It remains possible that requiring children to directly memorize basic addition facts may yield positive short-term effects. However, exclusively relying on rote learning as the foundation for teaching addition may not be optimal in the classroom. Placing too much emphasis on rote memorization at early stages may prevent children from practicing counting strategies, thereby impeding the emergence of automatized counting. Since automatized counting procedures can be faster than retrieval, children could be deprived of the opportunity to use the most efficient strategy. If retrieval fails, they will not have fast alternative strategies and will need to engage in conscious and resource-demanding counting or decomposition procedures. The attentional resources required to solve addition problems using cognitively demanding reconstructive strategies cannot be allocated to other critical aspects of tasks, such as interpreting word problems or understanding concepts in geometry or algebra.

If automatized counting is indeed a key mechanism in arithmetic fluency, activities involving counting, particularly games where children move pieces linearly on a board based on dice rolls or card draws, might be especially effective in promoting counting automatization. Such games have been shown to help children develop better understanding of number magnitude (Siegler & Ramani, 2008) and strengthen early arithmetic skills (Elofsson et al., 2016). Within the automatized counting framework, the efficiency of board games is readily explained because the linear movements on the board mirror the spatial navigation along a mental number line used by experts to solve addition problems. Other types of counting activities, such as finger counting, have also been shown to be effective in improving young children's performance on addition tasks (Poletti et al., 2025). Representing the operands of addition problems on fingers and counting all raised fingers could reinforce basic counting procedures that are candidates for automatization later during development. Other educational practices, such as solving addition problems through decomposition and composition, or by skip-counting (e.g., by 2 s or 3 s), provide alternatives to rote learning (e.g., Baroody, 2006; Fuson & Secada, 1986). In our view, these approaches may often be preferable because they allow children to engage in one-to-one counting indirectly, through derived strategies and micro-procedures, while maintaining a meaningful understanding of numbers. Of course, we cannot claim that the counting component of these activities directly drives or accelerates counting automatization, as they could also support retrieval. Rather, we emphasize that the positive effects of activities involving explicit counting are particularly well explained by an automatized counting framework.

Clinical implications

Our model may have implications for understanding mathematical learning disability (MLD). MLD is a broad term encompassing difficulties in acquiring mathematical competence that can arise from multiple causes, including specific deficits in symbolic or non-symbolic numerical processing as well as domain-general cognitive limitations (Butterworth, 2010; Geary, 1993). A frequent deficit is difficulty in automatizing simple arithmetic facts (Geary et al., 2012). Children with MLD rely more heavily on covert counting procedures than typically developing children, even for simple problems. In the literature, this apparent inability to transition away from covert counting is often attributed to limitations in working memory that would interfere with the formation of stable associations between problems and solutions when children use demanding counting strategies at the beginning of learning (Geary & Hoard, 2005). This is consistent with the finding that children with MLD eventually exhibit greater difficulties for non-tie than tie problems, the latter being less reliant on working memory when covert counting is used at the beginning of learning (Bagnoud et al., 2021).

However, the idea that simple addition problems may be solved through automatized counting procedures alongside fact retrieval may have implications for our understanding of arithmetic difficulties in MLD. Rather than necessarily reflecting a failure to associate arithmetic facts in long-term memory, the persistent use of demanding counting procedures in MLD may also indicate an impairment in the automatization of these procedures. This would not only be consistent with the dual process architecture proposed earlier, but also with the idea that MLD may involve multiple deficits, including both procedural difficulties and retrieval difficulties (Geary, 1993). That is, we hypothesize that some children with MLD may experience impaired automatization of counting procedures while others may experience weaker formation or retrieval of associations between operands and answers, contributing to the well-documented heterogeneity of MLD (Geary, 2004).

A potential deficit in automatized counting would be consistent with the difficulties experienced by children with MLD with non-tie addition problems (especially small problems) (Bagnoud et al., 2021). It would also be consistent with research suggesting that children with MLD may exhibit general procedural learning difficulties (Evans & Ullman, 2016). Indeed, the comorbidity between MLD and dyslexia may also be explained (to some extent) by common procedural learning deficits, as these have also been reported in dyslexia (Lum et al., 2013). However, it is important to note that procedural learning is not a unitary construct, and automatization of different types of procedures (e.g., phonological vs. numerical) may rely on partially dissociable neural circuits. Whether children with MLD show broader procedural deficits across multiple domains, or whether these deficits are specific to arithmetic procedures, remains an important question for future research.

This reconsideration of MLD as involving a difficulty in procedural automatization (as well as with retrieval of facts) may have implications for intervention approaches, which might incorporate structured practice of counting procedures as well as rote memorization. It is also important to acknowledge that the difficulty in automatizing counting procedures may arise from multiple underlying causes, including deficits in spatial-numerical processing (Huber et al., 2015), limitations in working memory affecting procedural learning (Szucs et al., 2013), or broader impairments in procedural memory systems (Evans & Ullman, 2016). In sum, the automatized counting model might provide new insights into the heterogeneous nature of MLD by highlighting the role of procedural automatization alongside (rather than instead of) retrieval processes, as well as its comorbidities with other learning disorders, such as dyslexia.

Summary

The automatized counting hypothesis has implications that extend beyond numerical cognition. Theoretically, it challenges the assumption that achieving fluency requires replacing procedural knowledge with associative retrieval, suggesting instead that procedural automatization may operate alongside associative retrieval and be fundamental to developing expertise in symbolic knowledge. This view may allow for a deeper understanding of the development of expertise, not only in arithmetic but also across domains of symbolic knowledge where systematic procedures are practiced extensively (e.g., reading, spelling). Educationally, the model questions the reliance on rote memorization in early math instruction, suggesting that extensive practice with counting procedures may be more effective for developing arithmetic fluency. Clinically, it helps reconceptualizing MLD as potentially involving deficits in both procedural automatization and memory formation, offering perspectives on intervention and potentially explaining comorbidities with disorders like dyslexia.

Conclusion

Here we have demonstrated that associative models alone cannot fully account for several critical findings in arithmetical cognition. Notably, addition and multiplication yield distinct behavioral and neuroimaging patterns that cannot be explained by models assuming that both operations rely on retrieval from long-term memory. Furthermore, associative models alone fail to explain the non-monotonic shape of the problem size effect. In contrast, the automatized counting theory provides a straightforward explanation: problems with operands up to 4 may predominantly be solved through one-by-one counting procedures, which may be faster than memory retrieval for these problems. Each additional unit requires one more counting step, leading to a linear increase in solution times for small problems. Due to cognitive limitations, fast and unconscious counting procedures may become less efficient for problems with operands larger than 4, where retrieval may become the most effective pathway.

We propose that the problem size effect appears in adults and older children for the same reason as in younger children: counting. According to Occam's razor, this developmental continuity (i.e., positing that counting becomes automatized rather than being replaced by an entirely different retrieval process) should be favored over retrieval-based explanations that require a qualitative shift

in mechanism. Associative models argue that retrieval times vary due to increased interference as problem size increases, yet must assume that tie-problems, also believed to be retrieved, constitute exceptions represented in separate neural networks. This claim lacks both parsimony and empirical support. In the automatized counting model, the interpretation is straightforward: small tie problems may primarily be retrieved from memory, while small non-tie problems may primarily be solved through automatized counting.

Furthermore, automatized counting procedures may operate as micro-procedures that may be used to solve other arithmetic operations and more complex addition problems. Therefore, our theoretical framework may extend beyond addition problems with operands up to 4. One example would be subtraction, which is often solved via addition (i.e., counting up, Barrouillet et al., 2008; Peters et al., 2010; Verschaffel et al., 2021). For instance, to solve $34 - 12$, individuals may count forward from 2 to 4 and from 1 to 3 and then combine these two intermediary results (2 and 2) to obtain the final result of 22. Automatized counting may be involved in such contexts. With respect to larger addition problems, these are frequently solved through decomposition (e.g., Lemaire & Arnaud, 2008). For instance, $53 + 44$ can be computed using a column procedure, in which the sub-addition $3 + 4$ may be solved via an automatized procedure prior to the completion of the calculation.

We are therefore confident that the automatized counting framework provides a solid foundation for further testing and extension, and that it is precise enough to generate additional specific predictions that can be contrasted with retrieval-based accounts of cognitive processing. For example, the boundary at 4 as the highest operand for which the counting procedure can become automatized is viewed as corresponding to the maximum number of elements that most adults can apprehend at a glance, paralleling the limits of subitizing. However, some individuals can subitize only up to three objects while others show a subitizing range extending to five (e.g., Leibovich-Raveh et al., 2018). Therefore, the range in automatized counting should vary accordingly. Individuals who can subitize up to five objects should be able to automatize counting up to $5 + 4$, leading to a break in the slope of solution times at 9. By contrast, individuals whose subitizing range is limited to three objects may only automatize counting up to $3 + 2$, with a corresponding break around 5. In contrast, because retrieval from memory is not constrained by subitizing limits, no change in the position of the break as a function of individuals' subitizing abilities would be predicted by retrieval models. Another way to test the automatized counting framework would be to relate individuals' slopes in the addition task to the speed of their memory scanning in a Sternberg task (Sternberg, 1966). Because automatized counting can be conceptualized as a form of serial scanning through a memorized sequence of numbers, the two measures should be correlated beyond general cognitive abilities. Again, this relationship would not be predicted by retrieval models. Finally, neuropsychological approaches can be particularly valuable for confronting retrieval and counting models, because they allow for the examination of how these processes are affected in individuals with specific cognitive impairments. For example, Parkinson's patients are particularly affected at the procedural level (e.g., Allain et al., 1995). The automatized counting theory would therefore predict greater difficulties for these patients with small addition than with small multiplication problems, whereas retrieval models would not predict such a dissociation. Conversely, children with agenesis of the corpus callosum are particularly vulnerable to interference during retrieval due to inhibition deficits (Soon et al., 2024). The automatized counting theory would predict specific difficulties in multiplication but not in addition for these children, whereas retrieval models would predict comparable difficulties across both tasks.

To conclude, we argue that counting, far from being abandoned in developing arithmetic fluency, remains a fundamental mechanism underlying skilled arithmetic performance. The idea that counting is an enduring foundation rather than a developmental stepping stone offers a developmentally parsimonious explanation for a range of empirical data. The automatized counting model has practical implications for education and clinical purposes. It also has broader theoretical implications for understanding skilled performance across cognitive domains. It challenges the conventional view that associations stored in long-term memory are the primary or sole mechanism underlying cognitive tasks that require fluency and automaticity. Instead, our model suggests that rapid procedures run in working memory can in some cases be more effective than retrieval from an extensive, interference-prone network. This distinction between procedural automatization and associative retrieval maps onto broader theoretical frameworks distinguishing procedural and declarative memory systems (Ullman, 2004). Specifically, our findings suggest that these systems may operate in parallel rather than one necessarily replacing the other during skill acquisition. This shift in perspective opens new avenues for understanding how humans process information automatically and efficiently in real-world situations, from symbolic processing (reading, spelling, calculation) to perceptual-motor skills (typing, musical performance) and wherever extensive practice of systematic procedures is central to learning.

CRediT authorship contribution statement

Jérôme Prado: Conceptualization, Writing – original draft, Writing – review & editing. **Catherine Thevenot:** Conceptualization, Writing – original draft, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

No data was used for the research described in the article.

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