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THEORETICAL/REVIEW

Spatial attention in mental arithmetic: A literature review and meta-analysis

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Abstract
We review the evidence for the conceptual association between arithmetic and space and quantify the effect size in meta-analyses. We focus on three effects: (a) the operational momentum effect (OME), which has been defined as participants' tendency to overestimate results of addition problems and underestimate results of subtraction problems; (b) the arithmetic cueing effect, in which arithmetic problems serve as spatial cues in target detection or temporal order judgment tasks; and (c) the associations between arithmetic and space observed with eye- and hand-tracking studies. The OME was consistently found in paradigms that provided the participants with numerical response alternatives. The OME shows a large effect size, driven by an underestimation during subtraction while addition was unbiased. In contrast, paradigms in which participants indicated their estimate by transcoding their final estimate to a spatial reference frame revealed no consistent OME. Arithmetic cueing studies show a reliable small to medium effect size, driven by a rightward bias for addition. Finally, eye- and hand-tracking studies point to replicable associations between arithmetic and eye or hand movements. To account for the complexity of the observed pattern, we introduce the Adaptive Pathways in Mental Arithmetic (APiMA) framework. The model accommodates central notions of numerical and arithmetic processing and helps identifying which pathway a given paradigm operates on. It proposes that the divergence between OME and arithmetic cueing studies comes from the predominant use of non-symbolic versus symbolic stimuli, respectively. Overall, our review and findings clearly support an association between arithmetic and spatial processing.

Keywords Spatial attention · Mental arithmetic · Representational momentum · Temporal order judgment · Target detection

Introduction

Spatial thinking has long been thought to play an important role in mathematics. This is obvious in domains such as geometry or measurement, which involve the explicit mapping of numbers to space. But a large body of evidence also indicates that numerical quantities in themselves may rely on spatial representations in the human mind (Hubbard et al., 2005; Toomarian & Hubbard, 2018). Specifically, a central theoretical framework for interpreting a range of effects in numerical cognition is that of the mental number line (MNL), according to which numerical magnitude is represented along a spatially oriented one-dimensional manifold. It has been argued that whenever we are processing a given number, its position on the MNL is activated. Noise in the cognitive system would lead to the concurrent coactivation of adjacent positions with an activation strength that decreases as numerical distance to the perceived number increases (Nieder & Dehaene, 2009). The MNL metaphor can provide a comprehensive explanation for a plethora of empirical findings, including hallmark effects in numerical cognition such as numerical distance (or ratio) and size effects, and can even account for congruency effects between the internally activated position of a number and object positions in physical space (Gianelli et al., 2012). Its explanatory power is also bolstered by the existence of number-sensitive neurons in the parietal and frontal cortices (Nieder, 2016). These neurons are topographically organized in a manner that reflects major principles of the MNL (Harvey et al.,

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2017), which supports its biological implementation at the neural level.

The idea that numbers are spatially organized along the MNL more generally suggests that the cultural achievement of mathematics might coopt neural mechanisms that have evolved for interacting with physical space, for example while planning our next saccade or guiding the movement of our hands (Hubbard et al., 2005). In a seminal paper, Hubbard et al. (2005) notably hypothesized that mental arithmetic can be conceptualized as attentional movements along the MNL, such that "when human participants compute additions or subtractions on numerical symbols, they should shift their attention to the left for subtraction problems, and to the right for addition problems" (p. 446, Hubbard et al., 2005). In other words, there might be space-arithmetic associations (SAA) much like there are associations between space and numbers (see also Fischer & Shaki, 2014).

Here, we review the relevant body of work conducted since that hypothesis was made and evaluate the strength of evidence for SAAs through the lens of three empirical phenomena: (a) the operational momentum effect (OME); (b) the arithmetic cueing effect; and (c) the attentional biases measured with eye- or hand-tracking during arithmetic calculation. For the former two phenomena, we amend our review by formal meta-analyses. We then present the currently prevailing theoretical accounts for SAAs and interpret the results of our review against this background before introducing the Adaptive Pathways in Mental Arithmetic (APiMA) framework that accommodates central notions of numerical and arithmetic processing.

Evidence for space-arithmetic associations (SAAs)

The operational momentum effect (OME)

Historically, the first main phenomenon suggesting the presence of SAAs is the *operational momentum effect* (OME), which involves the study of patterns of errors made by participants while they add or subtract approximate quantities. The OME describes a systematic bias in evaluating and estimating the outcomes of arithmetic problems. Specifically, for a given arithmetic outcome that is identical in addition and subtraction (e.g., 9 + 7 = 16 and 24 – 8 = 16), participants prefer larger outcomes for addition as compared to subtraction problems. For example, when both operands and response alternatives are presented as sets of dots, participants are more likely to accept an outcome such as 21 as the outcome of the problem 9 + 7 compared to the actual outcome (16) (McCrink et al., 2007). However, for the corresponding subtraction problem 24 – 8, participants would be more prone to accept an outcome such as 10 as compared to the actual outcome (McCrink et al., 2007).

While a consensus exists concerning the basic finding described above (i.e., the moderating role of the arithmetic operation on performance), the definition of the OME remains a matter of debate. Initially, the effect was defined as the overestimation of addition results and the underestimation of subtraction results as compared to the actual outcome (McCrink et al., 2007). Later studies, such as Knops, Viarouge, & Dehaene (2009b) (whose paradigm is depicted on Fig. 1A), defined the OME as the relative difference between addition and subtraction estimates that can both be subject to an overall bias (e.g., underestimation in the context of non-symbolic arithmetic). Here, we adopt the more lenient definition of the OME: We consider that the relative difference between estimates from different arithmetic operations such as addition and subtraction is the minimally necessary element that would reflect a moderating role of the arithmetic operation on performance.

Initially, the OME was described by McCrink et al. (2007) as a bias in approximate non-symbolic arithmetic (using dot patterns as stimuli). However, the effect was quickly found to generalize to symbolic notations as well (Knops, Viarouge et al., 2009b; Pinhas & Fischer, 2008), whether operations are matched with respect to operands or results (Knops, Viarouge, & Dehaene, 2009b). Because symbolic arithmetic has long been thought to involve verbal retrieval of answers from memory (Ashcraft & Fierman, 1982; Campbell & Xue, 2001; Seyler et al., 2003), the presence of an OME with symbolic notations was interpreted in a dual-process approach of mental arithmetic where the exact and verbally mediated retrieval process is paralleled by an arithmetic approximation process that operates on the MNL. While this dual-process may in theory be present in both symbolic and non-symbolic arithmetic, the OME is stronger with non-symbolic stimuli because these are associated with exact verbal retrieval processes to a much lesser extent (e.g., Knops, Viarouge, et al., 2009b). Hence, the exact verbal-numerical processes might reduce and overshadow the effects emerging from the approximate (spatial-attentional) processes.

Consistent with Hubbard and colleagues' (Hubbard et al., 2005) hypothesis that mental arithmetic might involve shifts of attention along the MNL (Hubbard et al., 2005), the OME has often been interpreted as reflecting a cognitive momentum that emerges from attentional processes. In other words, attention would mediate a displacement along a spatially oriented mental number representation (Knops, Thirion, Hubbard, Michel, & Dehaene, 2009a; Knops, Viarouge et al., 2009b; McCrink et al., 2007). Yet, some findings are not necessarily easily accounted by this hypothesis of attentional shifts along an MNL. For example, this hypothesis would predict that the size of the displacement (i.e., the numerical

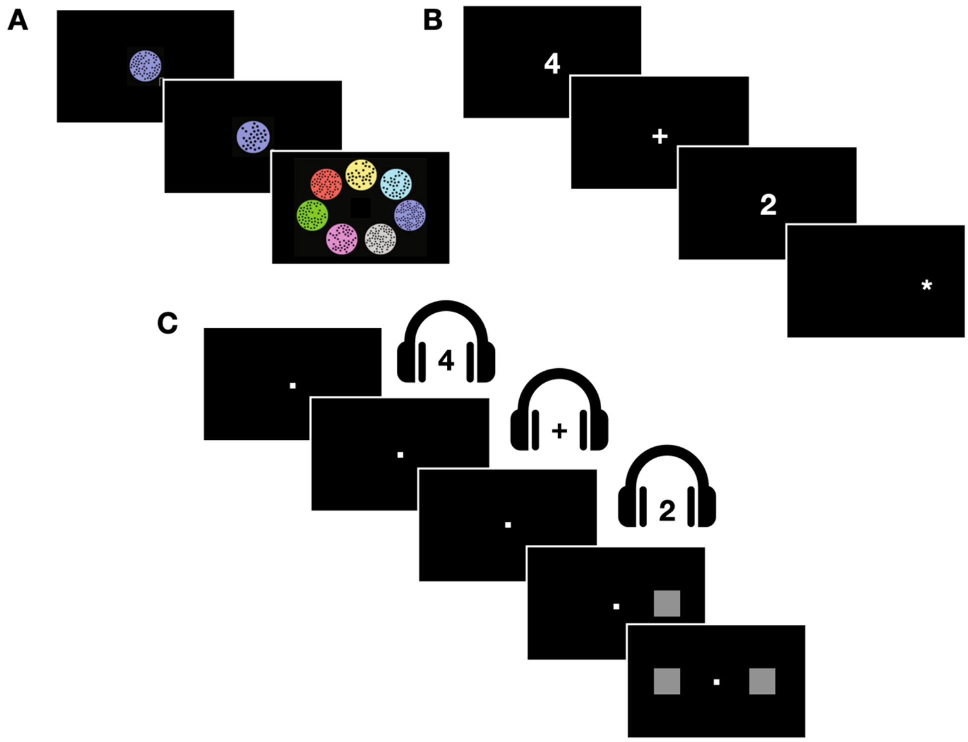

Fig. 1 Sample trials of paradigms used to study space-arithmetic associations (SAAs). (**A**) Operational momentum task (from Knops, Viarouge et al., 2009b). Participants are sequentially presented with two quantities and have to estimate the outcome of their addition by choosing among different options. (**B**) Arithmetic cueing task (from Masson & Pesenti, 2014). After solving an addition problem, participants have to detect a target in either the left or right visual field. (**C**) Temporal order judgment (TOJ) task (from Glaser & Knops, 2020). After solving an addition problem (presented auditorily), participants have to judge which of two lateralized targets is presented first

magnitude of the second operand) might modulate the OME. However, the size of the first or the second operand does not appear to be systematically linked to the size of the OME (Charras et al., 2014; Knops, Viarouge, et al., 2009b). In contrast, the OME increases with the arithmetic outcome (i.e., the problem size; Knops, Viarouge, et al., 2009b) and an overall underestimation is observed in tie problems (i.e., where both operands are identical) (Charras et al., 2014).

Attentional resources also appear to modulate the OME in a way that is not necessarily consistent with the idea of attentional shifts. For example, using a dual-task design, McCrink and Hubbard (2017) hypothesized that the OME would be reduced when less attentional resources are available. McCrink and Hubbard compared the amount of operational momentum in a baseline condition with two conditions in which participants had to concurrently process the non-symbolic arithmetic operands and monitor whether simple (color patches) or complex visual stimuli ('greebles'; Gauthier & Tarr, 1997) would be presented repeatedly. Surprisingly, compared to baseline, they observed an increased OME in addition trials in both simple and complex dual-task conditions while the OME in subtraction was unaffected by the concomitant task. McCrink and Hubbard interpreted these results as being at odds with the attentional shift hypothesis, as they predicted decreased OME in the dual-task conditions. Rather, they argue, the results are in line with the idea that the OME is a special case of representational momentum effect, which in turn is increased by heuristics. With reduced attentional resources available, heuristics such as "addition leads to more, subtraction leads to less" prevail and lead to an increased OME. Note that, while this may explain the increased OME in addition, it does not explain the differential impact of the dual-task load on OME under the two arithmetic operations (i.e., the absence of increased OME in subtraction).

As mentioned above, the OME can be minimally defined as the relative difference between estimates from addition and subtraction. To formally explore whether the OME is driven by an overestimation of addition or an underestimation of subtraction, we included relevant studies in a meta-analysis. Studies were identified from the PubMed database using the search terms: "Operational AND Momentum AND Arithmetic." This search identified 31 manuscripts. A second search using the terms "operational momentum AND numerical cognition" yielded 23 results. This was amended by a Pubmed search for articles that cited McCrink et al. (2007), Knops, Thirion et al., 2009a, Knops, Viarouge et al., 2009b, or Pinhas

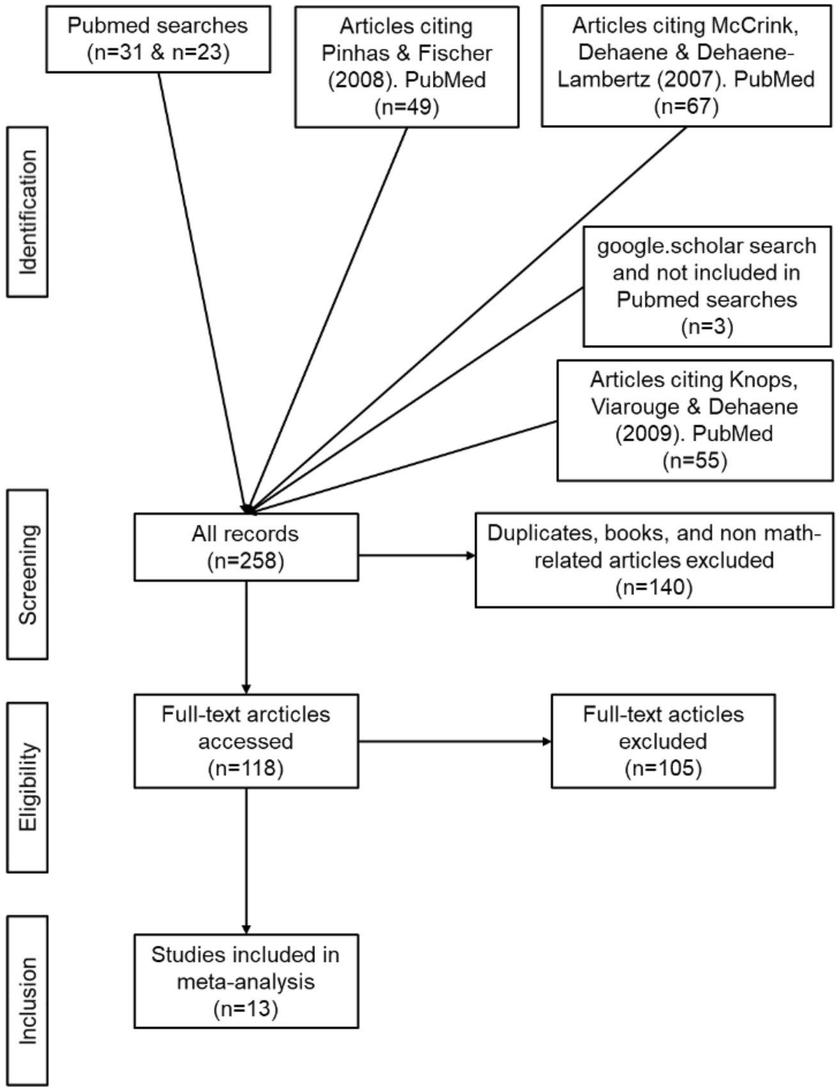

Fig. 2 Flowchart of literature search, identification of eligible articles for the meta-analysis of the operational momentum effect

and Fischer (2008), which yielded 67, 55, and 49 results, respectively. Using an ancestral search on www.scholar.google.com, we identified another three studies that were not listed in the PubMed search. After removing duplicates, we identified a total of 118 manuscripts. Next, we excluded all non-empirical reports, studies that investigated children, non-human participants, report results of computational simulations, or investigated other arithmetic operations than addition or subtraction. We excluded studies with non-canonical orientations (diverging from left-to-right reading direction in Western cultures) of the response dimension since it was unclear how to code these results with respect to right and left sided biases. Finally, we included only those studies that reported the mean numerical deviation between correct outcomes and participants' choices or between operations (e.g., focused on reaction time differences instead) in order to quantify the amount of the OME (see flowchart in Fig. 2).

With these inclusion criteria, we identified 13 studies investigating the OME, seven using a direct evaluation or production of the internally generated outcome and six adopting a transcoding approach (where the internally generated outcome had to be transcoded into a position on a line, see below).

For a formal evaluation of the OME, we entered effect sizes (Cohen's d) from seven studies using a direct evaluation or production of the internally generated outcome into the analysis using the MAJOR package in the Jamovi 2.3.19.0 software (see Fig. 3). The overall effect size across the studies was calculated based on a weighted average accounting for differences in statistical power between studies. A random-effects model was used to account for the

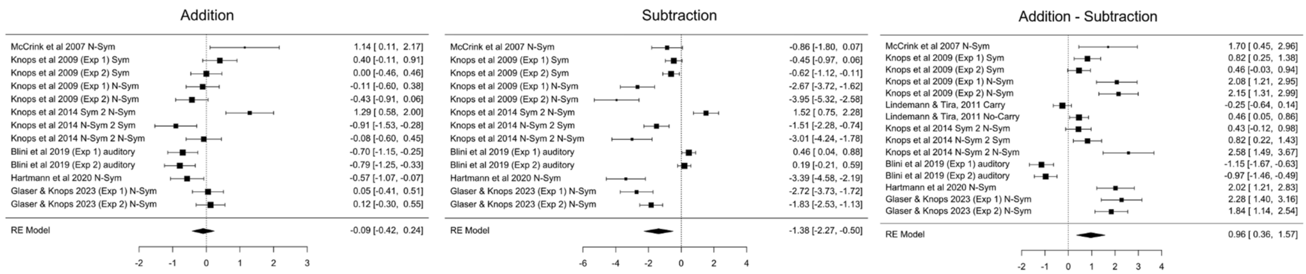

Fig. 3 Forest plots of the operational momentum effect (OME) for addition and subtraction problems (as well as a comparison between operations). The square boxes show the effect size in each study. The size of each box reflects the sample size and error bar the 95% confidence interval. The midline of the diamond indicated the mean pooled effect size and the diamond's width the 95% confidence interval. Positive (negative) effect sizes indicate an overestimation (underestimation)

possibility of systematic variation across studies. For each measure was calculated the ninety-five percent confidence intervals (CI) as well as the Z and p values corresponding to the estimate of the overall effect size. Beyond testing effect sizes for significant differences against zero, MAJOR also uses a "two one-sided test" (TOST) of equivalence that tests whether "the observed effect falls within the equivalence bounds and is close enough to zero to be practically equivalent" (Lakens, 2017, p. 355). A list of identified studies can be found at https://osf.io/download/6606ba5358fa4908a2e4ecf2/?view_only=144d8aab62884d608d3762f9b0bdd06d.

Overall, we found no overestimation in addition trials, with a mean effect size of d = 0.09 ($CI_{95\%}$= [-0.42 – 0.24], Z = 0.56, p = 0.58). Given equivalence bounds of -0.50 and 0.50, the equivalence test was significant (Z = 2.43, p = 0.008), suggesting that the observed effect is statistically not different from zero and statistically equivalent to zero. For subtraction, however, we observed a significant underestimation, with a mean effect size of d = -1.38 ($CI_{95\%}$= [-2.27 – -0.50], Z = -3.06, p = 0.002). Consequently, an overall significant OME was observed when comparing addition to subtraction with a mean effect size d = 0.96 ($CI_{95\%}$= [0.36 – 1.57], Z = 3.11, p = .002). Taken together, this quantitative meta-analysis indicates a reliable OME across studies. However, the effect appears to be mainly driven by an underestimation of subtraction problems, while addition overall leads to unbiased estimates.

To separate arithmetic processing from the impact of the arithmetic operator, a number of studies have examined the OME with zero as a second operand. In such so-called *zero-problems* (e.g., 3 + 0 = ?; 7 – 0 = ?), a regular OME has been observed both when participants produce the non-symbolic outcomes (Lindemann & Tira, 2011) and when they indicate the position of the outcome on a left-to-right oriented labeled number line (Pinhas & Fischer, 2008; Shaki et al., 2018). These results have been interpreted as evidence against the attentional shift explanation of the OME, since no attentional displacement would be required with zero as second operand. Yet, when participants are asked to transcode the estimated outcome to a line length, no statistically significant difference between addition and subtraction is observed (Mioni et al., 2021).

To elucidate the temporal and contextual malleability of the mental number representation, some researchers asked participants to indicate where the result of an arithmetic problem would be positioned on a labeled line. In these studies, the orientation (i.e., increasing numerical magnitude from left to right or from right to left) of the labeled number line was manipulated and pitted against the hypothesized left-to-right orientation of the MNL in long-term memory. The results demonstrated that the bias induced by addition is not consistently driving responses to the right side of space (Klein et al., 2014; Pinhas et al., 2015). For example, both Klein et al. (2014) and Pinhas et al. (2015) have shown that when the larger numbers are on the left side (and the smaller numbers on the right side) of the external response dimension, responses are biased towards the larger number. Note, however, that Pinhas et al. (2015) found that this effect was moderated by the type of arithmetic problem. That is, with non-zero problems (i.e., where none of the operands is zero) with either 4 or 6 as a result, the authors observed a reverse OME for 6 (i.e., addition was oriented further to the left compared to subtraction) and a regular OME for 4 (i.e., addition led to responses that were further to the right compared to subtraction). For zero problems, however, responses for addition problems were displaced to the left side compared to responses in subtraction problems. Overall, then, there is some evidence that the layout of the mental number representation during OM tasks is relatively flexible and task-dependent. This is reminiscent of the discussion on whether or not the spatial orientation of the mental number line is systematically oriented from left to right in long-term memory, for example as a result of cultural conventions such as reading and writing direction (e.g., Shaki et al., 2009), or whether it is constructed in a task-dependent manner in working memory (Fias et al., 2011; van Dijck & Fias, 2011). Interestingly, in Pinhas

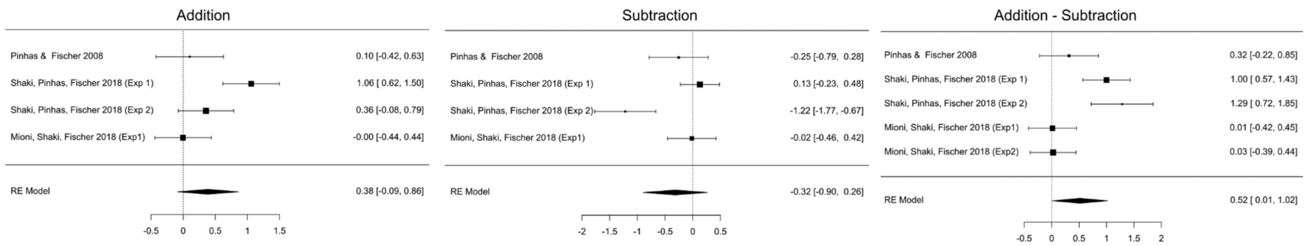

Fig. 4 Forest plots of the operational momentum effect (OME) measured in pointing tasks with zero problems for addition and subtraction problems (as well as a comparison between operations). The square boxes show the effect size in each study. The size of each box reflects the sample size and error bar the 95% confidence interval. The midline of the diamond indicated the mean pooled effect size and the diamond's width the 95% confidence interval. Positive (negative) effect sizes indicate an overestimation (underestimation)

et al. (2015), the same participant sample who showed a flexible OME also exhibited a standard SNARC effect both before and after being presented with the right-to-left-oriented number line in the OM task. Together with earlier findings showing that the SNARC effect, too, is task- and context-dependent (Bächtold et al., 1998), these results underline the idea that the spatial layout of the internal representations that are deployed during number processing and mental arithmetic is highly flexible and adaptive to situational (spatial layout of external response or stimulus space) and cultural factors (e.g., reading and writing habits).

Besides paradigms where participants choose the preferred outcome amongst several alternatives, the OME, and in particular the OME in zero problems, has also been tested in a paradigm where participants have to transcode the internally generated results. Two variants of this paradigm can be found in the literature. In one variant, participants transcode the result into a spatial position which they indicate on a labeled number line. In a second variant, participants produce lines whose length corresponds to the numerical magnitude of the target (e.g., the internally generated result). We now consider these studies in a meta-analysis that is separate from the one reported above for two reasons. First, this response mode might involve an additional transcoding process, which may be a source of additional biases and differences with the classic OME paradigms. For example, for number ranges that are unfamiliar to the participants, a variety of individual strategies have been identified that deviate from a linear mapping (Landy et al., 2013, 2017) Second, the use of an explicit and external spatial representation may reinforce the association between numbers and space as opposed to the implicit (or at least un-ordered) character of the response modality in tasks that do not rely on number-to-space mappings. This implicit-explicit distinction also figures amongst the main organizational principles of a recent taxonomy of spatial-numerical associations (Cipora et al., 2015). Third, the results of these studies have been used to establish zero problems as the gold standard for measuring the OME (see above). Note that we only included studies that adopted a canonical orientation of the response metric (i.e., small numbers on the left and large numbers on the right).

When computing the mean effect size in studies that required participants to transcode the outcome of a given problem to a position on a labeled line or a line length, the OME in zero problems only emerges when directly contrasting addition to subtraction (mean $d = 0.51$; $CI_{95\%}$= [0.01 – 1.02], $Z = 2.0$; $p = .046$) (see Fig. 4). This paradigm neither yielded strong evidence for significant effects in zero addition (mean $d = 0.38$; $CI_{95\%}$ = [-0.09 - 0.86], $Z = 1.59$, $p = .111$) nor subtraction (mean $d = -0.32$; CI: -0.9 – 0.26) problems against baseline. However, in both cases, non-significant equivalence tests with boundaries [-.5, .5] suggested that the observed effects are not equivalent to zero (addition: $Z = -0.479$, $p = .319$; subtraction = 0.614, $p = .270$). Hence, the overall picture that emerges remains ambiguous, most likely due to the limited number of studies included (n = 4).

A similar picture emerged for pointing experiments with operands that are different from zero (see Fig. 5). For addition (mean d = - 0.39; $CI_{95\%}$ = [-0.70 - -0.08], $Z = -2.463$, $p = .014$), a significant underestimation was observed. Neither subtraction (mean d = - 0.21; $CI_{95\%}$ = -0.86 – 0.43; $Z = -.649$, $p = .516$) nor the comparison between addition and subtraction (mean $d = -0.06$; $CI_{95\%}$ = -0.57 – 0.45; $Z = -0.225$, $p = .822$) yielded significant results. For subtraction, the equivalence test was non-significant ($Z = 0.865$, $p = .193$), allowing no statistical conclusion. For the direct comparison of addition and subtraction (corresponding to the minimal definition of the OME) the equivalence test was significant ($Z = 1.692$, $p = .045$, with the boundaries [-.5, .5], allowing the conclusion that the effect is statistically equivalent to zero. The data from studies involving a spatial transcoding are hence overall less conclusive than in the previous meta-analysis and only show an OME in zero problems (when comparing addition and subtraction) and an inverse OME in nonzero addition problems. Since neither of the arithmetic operations yields significant effects against baseline, the question of what drives the OME in studies that involve a transformation during or before the responses

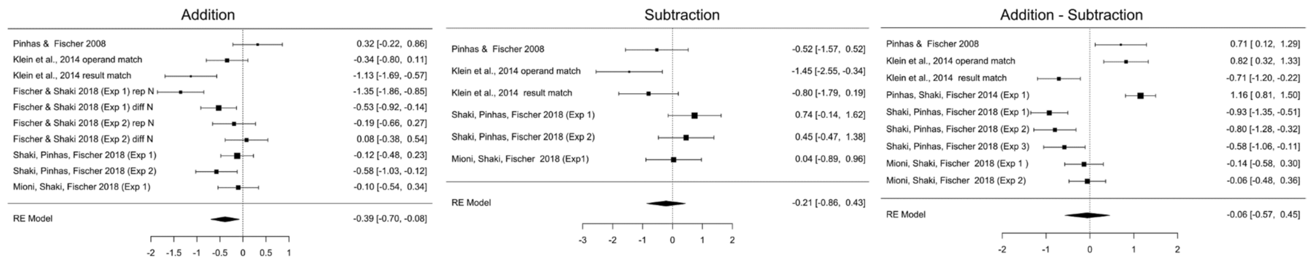

Fig. 5 Forest plots of the operational momentum effect (OME) measured in pointing tasks with nonzero problems for addition and subtraction problems (as well as a comparison between operations). The square boxes show the effect size in each study. The size of each box reflects the sample size and error bar the 95% confidence interval. The midline of the diamond indicated the mean pooled effect size and the diamond's width the 95% confidence interval. Positive (negative) effect sizes indicate an overestimation (underestimation)

remains open. One explanation for this lack of consistency may be that the paradigm in and of itself requires an additional mapping from an internally generated outcome to an external reference frame (position on a line of a certain length). This may induce additional biases and involve strategies such as visual anchoring on salient reference points (e.g., the middle).

Arithmetic cueing

In arithmetic cueing tasks, arithmetic problems are designed to serve as implicit spatial cues for subsequent lateralized targets presented in either the left or right visual field. Such paradigms are largely inspired by previous research on an effect that is sometimes called "attentional SNARC" (Fischer et al., 2003). By asking adult participants to detect lateralized targets briefly presented after a non-informative central digit cue, Fischer and colleagues showed an interaction between the size of the digit (relatively small or relatively large) and the side of presentation of the target (left or right). That is, the presentation of task-irrelevant digits smaller than five facilitated the detection of subsequent targets in the left visual field whereas the presentation of task-irrelevant digits larger than five facilitated the detection of subsequent targets in the right visual field. Fischer et al.'s results have been influential in the field because they support the idea that the mental number line is spatially organized from left to right, though there is a debate regarding the replicability of these findings. On the one hand, recent studies exploiting eye-tracking data such as gaze position (Loetscher et al., 2010; Myachykov et al., 2016; Salvaggio et al., 2019) or pupil dilatation (Salvaggio, Andres, Zénon, & Masson, 2022a) seem to support the idea that number cues do induce shifts of spatial attention. On the other hand, both a multi-site replication attempt of the study (Colling et al., 2020; Fischer et al., 2020) and recent behavioral studies investigating manual reaction times (as in the original paradigm) have failed to show that attentional deflections are induced by number cues (Galarraga et al., 2022; Hesselmann & Knops, 2023). The depth with which the central number cues needs to be processed has been identified as a modulating factor for the observation of attentional shifts, recently (Shaki & Fischer, 2024). While the exact conditions under which numbers potentially shift attention (or not) are still elusive, the idea inspired subsequent research on attentional deflections during mental arithmetic where depth of semantic processing does not play a huge role.

Masson and Pesenti (2014) were the first to adapt the (manual) attentional SNARC task in the domain of mental arithmetic. In their version of the task, participants also had to detect targets in the left or right visual field. However, a visually (and sequentially) presented arithmetic problem (addition or subtraction) that participants had to solve replaced the digit cue of Fischer et al.'s experiment (see Fig. 1B). The hypothesis was that associations between arithmetic operations and representational number space would translate into an interaction between type of operation and target position in physical space. That is, if subtraction is associated with the left side of representational space, left targets should be detected faster than right targets when they follow subtraction problems. Likewise, left targets should be detected faster when they follow subtraction compared to addition problems. In contrast, if addition is associated with the right side of representational space, right targets should be detected faster than left targets when they follow addition problems and right targets should be detected faster when they follow addition than subtraction problems. Overall, Masson and Pesenti (2014) reported the expected interaction between type of operation and target position in two experiments, with single-digit subtraction speeding up the detection of left (as compared to right) targets and double-digit addition speeding up the detection of right (as compared to left) targets.

Masson and Pesenti's (2014) results inspired a number of studies that subsequently investigated the relation between arithmetic processing and spatial attention in a more explicit manner than with the OME. These studies are reviewed in what follows. For the sake of the present

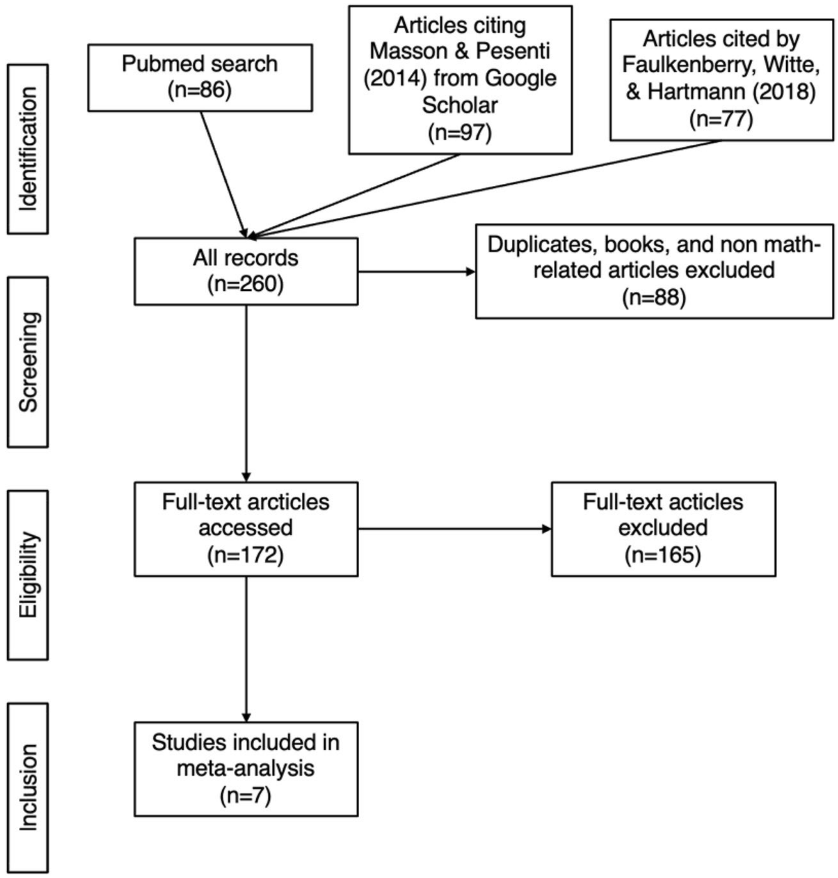

Fig. 6 Flowchart of literature search, identification of eligible articles for the meta-analysis of the arithmetic cueing effect

review, studies were identified from the PubMed database using the following search terms: "((arithmetic [ot]) OR (addition [ot]) OR (subtraction [ot])) AND (space [ot] OR attention [ot] OR attentional [ot] OR shift [ot] OR bias [ot]) AND (journalarticle [Filter])". This was completed by two additional searches. First, we used Google Scholar to conduct a systematic search for articles that cited Masson and Pesenti (2014). Second, we conducted an ancestral search based on the references of a review of eye and hand tracking studies in numerical cognition (Faulkenberry et al., 2018). The search from PubMed returned 86 papers, while the search from Google Scholar returned 97 papers and the ancestral search returned 77 papers. After removing duplicates, books, and articles that were unrelated to math cognition, 172 papers were screened (see flowchart in Fig. 6). Papers are mentioned here if they (a) are not review or meta-analysis articles; (b) involve adult participants; (c) do not involve the OME; and (d) explicitly investigate the relation between arithmetic processing and spatial attention using cueing paradigms, eye-tracking, hand-tracking, lesion studies, and neuroimaging. A list of identified studies can be found at https://osf.io/fd7c3?view_only=144d8aab62%20243%20884d608d3762f9b0bdd06d.

Out of all the studies identified in our systematic search, seven studies employed variations of Masson and Pesenti's arithmetic cueing task, using either visual or auditory presentations of the arithmetic problems (Campbell et al., 2021; D'Ascenzo et al., 2020; Liu, Cai, Verguts, & Chen, 2017a; Liu, Verguts, Li, Ling, & Chen, 2017b; Masson et al., 2018; Mathieu et al., 2016; Zhu et al., 2018). Liu, Cai et al. (2017a), for example, reported associations between different stages of arithmetic processing and spatial positions. Specifically, the authors observed a leftward advantage for subtraction and a rightward advantage for addition when targets were presented after the second operand or the result. This is consistent with the idea that these biases reflect attentional shifts elicited by calculation. There was also a rightward advantage before the onset of the second operand in addition problems, i.e., when the target directly followed the '+' sign. However, no association was observed after the '+' sign when it was not preceded by the first operand. This suggests that arithmetic operators may be associated with attentional biases during mental arithmetic if they

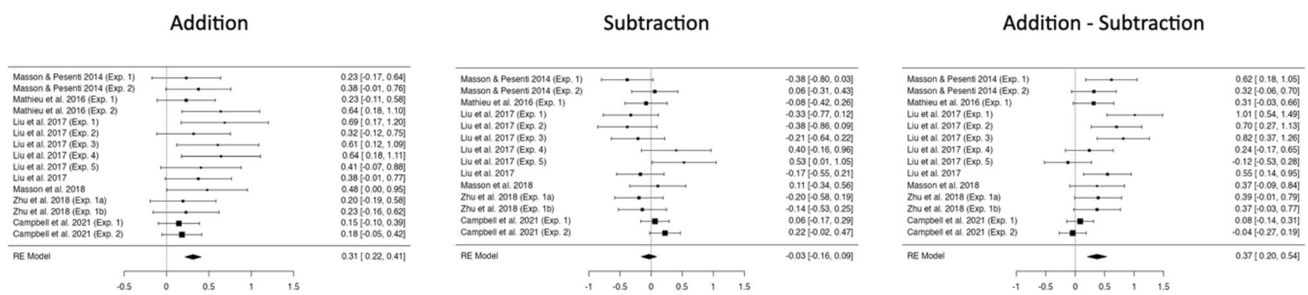

Fig. 7 Forest plots of the lateralized effects obtained in arithmetic cueing studies involving target detection along the horizontal dimension for addition and subtraction problems (as well as a comparison between operations). The square boxes show the effect size for the difference between left target and right target in each study. In addition, positive effect sizes indicate a cueing advantage for items on the right side; in subtraction, negative effect sizes indicate a cueing advantage for items on the left size. The size of each box reflects the sample size and error bar the 95% confidence interval. The diamond reflects the pooled effect size and the width of the 95% confidence interval

come after an operand. Liu, Verguts et al. (2017b) further presented targets in the vertical dimension, but could not find any association in that specific study (which conflicts with eye-tracking studies reviewed later). As another example, both Mathieu et al. (2016) and Campbell et al. (2021) used a version of the arithmetic cueing task in which the second operand was presented either to the left or to the right side of space, essentially acting as the target of Masson and Pesenti's task. To the extent that addition problems were intermixed with subtraction problems (thereby making the operator maximally relevant, see Prado & Thevenot, 2021), both studies reported that addition problems were solved faster when the second operand appeared to the right than to the left side. Evidence for an association between subtraction and left targets, however, was not found in Campbell et al. (2021).

Taken together, a qualitative review of arithmetic cueing studies appears to suggest stronger associations between addition and right-lateralized targets than between subtraction and left-lateralized targets. To formally assess whether this is the case, we extracted from the seven studies described above the effect sizes (Cohen's d) associated with detecting a left versus a right target either after or at different points during a subtraction and after an addition problem.[1] Note that some of these studies included different experiments (Campbell et al., 2021; Liu, Cai et al., 2017a; Masson & Pesenti, 2014; Zhu et al., 2018). Effect sizes were entered in a quantitative meta-analysis using the MAJOR package in the Jamovi 2.3.19.0 software, using the same protocol as for the OME (see Fig. 7). Results show that addition problems are indeed associated with faster detection of right than left targets, with a mean effect size of $d = 0.31$ ($CI_{95\%}$= [0.22 – 0.41], $Z = 6.38$, $p < 0.001$). In contrast, subtraction problems are not associated with faster detection of left than right targets across these studies (d = -0.03, $CI_{95\%}$= [-0.16 – 0.09], Z = -0.52, p = 0.60). Given equivalence bounds of -0.50 and 0.50, the equivalence test was significant ($Z = 7.30$, $p < 0.001$), indicating that the observed effect was statistically not different from zero and statistically equivalent to zero. Together, an overall significant difference was observed when comparing addition to subtraction, with a mean effect size of d = 0.37 ($CI_{95\%}$= [0.20 – 0.54], Z = 4.35, p < 0.001).

Recent studies have also embedded temporal order judgments (TOJs) into arithmetic cueing tasks to probe the relation between arithmetic and spatial processing (see Fig. 2C). TOJs involve the presentation of two lateralized targets with different stimulus onset asynchronies (SOAs) (Casarotti et al., 2007). Participants are typically asked to indicate which target is presented first. Although the probability of judging which target appears first clearly depends on the SOA, that judgment is also influenced by the location of spatial attention. The TOJ paradigm builds on a long-standing stance in experimental psychology that goes back to Titchener (1908) and is known as the prior-entry hypothesis: "the object of attention comes to consciousness more quickly than the objects which we are not attending to" (Titchener, 1908, p. 251). The target that is in the focus of attention enters the cognitive system first. This is even the case if the attended target is lagging behind in time. For example, directing attention toward the left visual field will bias participants to judge left targets as appearing earlier than right targets even if the SOA is null or if left targets are presented slightly after right targets. By presenting TOJs after asking participants to solve subtraction or addition

[1] Note that we included in this meta-analysis experiments with a variety of experimental parameters (e.g., different stimulus onset asynchronies (SOAs), different measurement time points). As reviewed above, there is evidence in the literature that arithmetic cueing effects depend on such parameters (e.g., Liu, Cai et al., 2017a). However, because our goal was to provide an estimate as objective as possible of the overall effect size associated with arithmetic cueing, we chose to include all conditions without discrimination. By doing so, we argue that our estimate provides a lower bound of the effect size that could be obtained under the most favorable conditions.

problems, it is then possible to probe the location of spatial attention after arithmetic problem-solving. Overall, TOJs tend to be more strongly biased towards the right side when targets followed from addition than subtraction problems (Andres et al., 2020; Glaser & Knops, 2020; Masson et al., 2020). That effect was observed across a range of problem sizes (Glaser & Knops, 2020) as well as with participants with different reading habits (Masson et al., 2020). However, a comparison of that effect against a baseline TOJ assessment revealed that it is more likely driven by addition being associated with the right side of space than by subtraction being associated with the left side of space (Glaser & Knops, 2020), in line with the meta-analysis described above. Interestingly, asking participants to judge which target appeared first or which target appeared last does not change participants' biases, which has been taken as evidence that the effect may depend more on semantic associations between operations and space than on movements along the mental number line (Andres et al., 2020).

Eye- and hand-tracking during calculation

Another approach that has been used to gather evidence for SAA involves measures of eye- and hand-tracking during arithmetic calculation. Most studies have used eye-tracking, which is one of the most straightforward measures of visual attention in cognitive psychology as it captures online gaze position during a task with excellent spatial and temporal accuracy (Kiefer et al., 2017). One of the earliest studies of this kind is from Werner and Raab (2014). The authors measured the gaze behavior of two groups of participants who were presented with both subtractive and additive problems involving the displacement of water between different recipients. The findings suggest a difference in gaze position between the groups, with a rightward bias for additive problems and a leftward bias for subtractive problems. Subsequent studies investigated gaze behavior of participants who were asked to solve symbolic arithmetic problems presented auditorily. These studies generally show evidence of systematic biases in gaze behavior that are dependent on the operation, though the timing, dimension, and in one case direction (Yu et al., 2016) of these shifts are not always consistent across experiments.

Combining eye-tracking with an arithmetic cueing design, Masson et al. (2018) measured eye position during different stages of an arithmetic problem while also asking participants to detect targets presented in either the left or right visual field after the problem had been solved. Although no gaze shift was measured from the onset of the first operand, operator, or second operand, a systematic rightward bias was observed in addition compared to subtraction between the offset of the second operand and the verbal response (i.e., the calculation stage). This finding was replicated in two recent studies by Blini et al. (2019) and Salvaggio, Masson, Zénon, and Andres (2022b), who also showed that this rightward movement is accompanied by an upward shift (note that Blini et al., 2019, also found a leftward and downward shift for subtraction). It is also consistent with the finding that participants' gaze is shifted rightward (and upward) when participants successively add numbers in a counting task (Hartmann et al. 2016). Altogether, these studies suggest that shifts of attention may manifest themselves through both the horizontal and vertical dimensions during the calculation stage of an arithmetic problem (see also Zhu et al., 2019).

Associations between arithmetic and space in the vertical dimension are consistent with an earlier study by Wiemers et al. (2014), who reported motion-arithmetic compatibility effects due to active body movements in both the horizontal and vertical dimensions, while eye movements pursuing the moving operands led to such effects only in the vertical dimension. It has been argued that vertical associations might differ from horizontal associations: While the former could be grounded in early-developing sensorimotor experience (e.g., moving upward when stacking objects), the latter may be particularly affected by later-developing cultural practices (e.g., reading and writing habits) (Blini et al., 2019; Hartmann, 2022; Wiemers et al., 2014). Future studies, however, are needed to substantiate this intriguing hypothesis.

Whether they are horizontal or vertical, late-occurring shifts of attention are consistent with the idea that they might reflect movements along the mental number line. However, there is also evidence of differences in eye position before the onset of the second operand. For example, both Salvaggio, Masson et al. (2022b) and Hartmann et al. (2015) found evidence for an operation-dependent bias in eye position even before the presentation of the second operand. That is, gaze was found to be moved upward (Hartmann et al., 2015) and rightward (Salvaggio, Masson et al., 2022b) after the presentation of the '+' sign (as compared to a '-' sign). This is consistent with arithmetic cueing studies that reported response biases induced by arithmetic operators in target detection tasks (as long as they are preceded by a first operand) (Liu, Cai et al., 2017a). Finally, evidence for a relation between findings from arithmetic cueing and eye-tracking studies is suggested by Masson et al. (2018). In that study, the more a participant's gaze was shifted rightward after an addition problem (as compared to a subtraction problem), the faster that participant was at detecting a target in the right visual field (as compared to the left visual field). Thus, online shifts of attention measured through gaze movements appear related to the response biases measured in classic arithmetic cueing tasks.

Tracking eye movement is perhaps the most direct way to measure spontaneous shifts of attention during mental arithmetic. However, shifts might manifest themselves through other effectors as well. For example, Marghetis et al. (2014) asked participants to select which of two numbers presented at the top left and top right corners of a screen is the correct solution of a single-digit addition or subtraction problem presented at the bottom of that screen. By tracking hand trajectories, the authors showed systematic rightward and leftward deflections when participants had to select the answer of an addition or a subtraction, respectively. Using a similar finger tracking methodology, Pinheiro-Chagas et al. (2017) asked participants to indicate the result of single-digit addition and subtraction problems on a number line. The findings give an interesting insight into the calculation process, with participants first pointing towards the largest operand before slowing deviating towards the result in a way that is proportional to the size of the smaller operand. While this pattern is clearly supportive of the idea that participants move along a mental number line when adding or subtracting, the study also shows the operator-dependent bias observed in many of the studies discussed above (with '+' signs attracting the finger to the right and '-' signs to the left).

Functional relevance of SAAs

By and large, all of the studies reviewed above investigate the presence of *associations* between arithmetic calculation and spatial processing. Therefore, such evidence is entirely correlational. Specifically, these studies do not make it possible to determine to what extent the attentional shifts that are observed during calculation are *necessary* to arithmetic processing or are simply a by-product of that processing with little functional relevance. Interestingly, a handful of studies suggest that attentional shifts may functionally matter for arithmetic calculation. Evidence for a causal role of attentional shifts during arithmetic processing comes from studies that examined arithmetic performance while attentional shifts are either impaired or manipulated.

For instance, an attention disorder that has a long history of investigation in neuropsychology is left unilateral neglect (Bisiach & Vallar, 2000). After a lesion in the right hemisphere (typically around the parietal cortex), these patients exhibit severe difficulties attending stimuli in the left visual field. By asking several of these patients to solve series of addition and subtraction problems, Dormal et al. (2014) showed that they were less accurate than control groups to solve large subtraction problems, whereas no difference was observed for large addition problems. In contrast, a patient with a rare right unilateral neglect (following from a left-hemisphere lesion) showed the reverse pattern, with specific impairment in solving addition but not subtraction problems (Masson, Pesenti, Coyette, Andres, & Dormal, 2017b). In other words, there appears to be a double dissociation between subtraction and addition problem solving in patients with left versus right unilateral neglect, demonstrating a causal role of spatial attention in arithmetic calculation.

Other studies have experimentally manipulated attentional shifts during arithmetic calculation in normal adults. In arithmetic cueing studies, for example, the target detection task follows the response given by participants for the arithmetic problem. Masson and Pesenti (2016) reversed that timeline, asking participants to pay attention to a flickering target between the second operand and the prompt to respond to the arithmetic problem. In a first experiment, Masson and Pesenti demonstrated that the flickering target captured attentional resources and slowed reaction times compared to a condition without flickering target presentation. In other words, the flickering targets acted as attention-capturing distractor in that study. In a second experiment with lateralized flickering targets, the authors observed an interaction between operation and side of the distractor, with subtraction being responded slower when distractors were on the left side and addition being responded slower when distractors were on the right side. Both Masson, Pesenti, and Dormal (2017b) and Blini et al., (2019) also manipulated attentional shifts during arithmetic calculation, this time by using optokinetic stimulation (OKS). OKS is a technique that uses moving visual displays to orient eye movements (and therefore attention) in the direction of the display movement. This allows researchers to manipulate the location of overt attention in a way that is either congruent or incongruent to the expected SAAs. Masson, Pesenti, and Dormal (2017b) found that shifting attention to the right facilitates addition problem solving as compared to shifting attention to the left (or not shifting attention), to the extent that these problems involve a carrying procedure. No reverse effect, however, was observed for subtraction problems. Using more complex problems and vertical as well as horizontal OKS, Blini et al. (2019) further showed that shifting attention downward reduced decade errors in subtraction problems (whereas shifting attention upward increased these errors). Therefore, studies do not consistently show similar causal effects of attentional shifts on arithmetic calculation, which may be due to differences in experimental procedures and materials. Nonetheless, several lines of evidence suggest that attentional shifts do have a causal effect on arithmetic calculation: experimentally manipulating shifts of attention appears to affect arithmetic performance (see also Hartmann, 2022; Masson & Pesenti, 2023).

Theoretical frameworks of SAAs

Several accounts of SAAs have been proposed over the years. Although most of these accounts have first attempted to explain the OME, some can be broadened to explain explicit associations between arithmetic and space (as measured by arithmetic cueing and eye- or hand-tracking paradigms). Below we briefly review some major theoretical frameworks conceptualizing SAAs.

Compression account

Relatively early on, number compression has been proposed as an explanation of the OME (Chen & Verguts, 2012; McCrink et al., 2007). According to this idea, the OME is caused by a systematically inaccurate decompression of presumably logarithmically compressed magnitude representations. As an extreme example, imagine that the addition of two operands (O1 + O2) would be computed on their logarithmically compressed internal representations (log(O1) + log(O2)). Since addition (subtraction) on the logarithmic scale corresponds to a multiplication (division) on a linear scale, this would lead to massive overestimations for addition (log(O1) + log(O2) = O1 × O2) and underestimations for subtraction. This mechanism has been implemented in a computational model of numerical cognition (Chen & Verguts, 2012), which was able to reproduce empirically observed performance patterns in addition and subtraction task with adults. Nonetheless, the compression account is not without challenges. First, it predicts that the amount of compression is linearly related to the size of the OME. In support of this notion, Knops, Thirion et al. (2009a), Knops, Viarouge et al. (2009b) reported that OME increases with the numerical magnitude of the outcome. However, when measuring the compression in a numerosity naming task, no correlation between OM and compression (or any other psychophysical property of the number system) was observed (Knops et al., 2014). Second, it has been argued that the numerical magnitude representation in children is subject to a more pronounced compression, which would suggest that the OME should be stronger in children compared to adults. However, the OME appears to emerge only around the age of 9 or 10 years and is absent or reversed in younger children (Pinheiro-Chagas et al., 2018). Third, and perhaps most importantly, the compression account is limited to the OME and does not readily explain other effects such as arithmetic cueing. As such, it is not a parsimonious theory of SAAs.

Attentional shift account

According to the attentional shift account, SAAs stem from attentional movements along the MNL. For example, it has been proposed that approximate mental arithmetic may be mediated by a dynamic interaction between positional codes on the MNL (place coding) and an attentional system that shifts the spatial focus to the left or right (Knops, Thirion et al., 2009a). At the neural level this may be instantiated in the functional interactions between areas along the intraparietal sulcus and posterior, superior parietal areas (Hubbard et al., 2005). This places mental arithmetic in the realm of dynamic updating processes of spatial coordinates in parietal cortex and stipulates that the efficiency of this system is linked with arithmetic performance. Due to the approximate nature of this process the shifts may "overshoot," leading to over- and underestimation in addition and subtraction, respectively. Not only does this account explain the OME, it also suggests a functional coupling between eye movements and arithmetic.

The attentional shift account has also been extended to the domain of exact symbolic arithmetic. For example, although classic models have long assumed that simple arithmetic problems (e.g., single-digit addition) were retrieved from memory in educated adults (Ashcraft, 1992; Campbell & Tarling, 1996), it has recently been proposed that these problems may also be solved using counting procedures that would become automatized over the course of learning and turn into mental scanning of the MNL (Barrouillet & Thevenot, 2013; Mathieu et al., 2016; Uittenhove et al., 2016). Such a fast mental scanning might potentially explain associations between arithmetic operations and space (Mathieu et al., 2016), though it has also been argued that this process might only be efficient enough to solve problems with small operands (Uittenhove et al., 2016). Nonetheless, this idea is consistent with those studies that have observed SAAs at the outset of problems, either immediately after the second operand (Liu, Cai et al., 2017a) or slightly after (Masson et al., 2018; Salvaggio, Masson et al., 2022b). This is also in keeping with Pinheiro-Chagas et al.'s (2017) findings that SAAs (measured through finger tracking along a number line) are proportional to the size of the second operand. To date, however, there is no evidence that either eye movements or magnitudes of spatial biases in arithmetic cueing studies relate to the size of the problem, as would be expected if these effects are due to movements along the MNL. The observation of an OME in zero problems (see above) has also challenged this explanation since no spatial displacement is involved when the second operand is zero. Yet, taken together, the attentional shift account provides a relatively parsimonious explanation of SAAs in the variety of paradigms reviewed above.

Heuristics account

A number of authors have proposed that heuristics are at the heart of SAAs. For instance, according to the "if adding, accept more" and "if subtracting, accept less" heuristics (McCrink et al., 2007; McCrink & Wynn, 2009), the OME is caused by the application of the general principle that for addition (subtraction) outcomes are accepted as long as they are larger (smaller) than the initial operand. McCrink and Hubbard (2017) recently proposed that the heuristics account and the attentional shift account might even belong to one single mechanism (heuristics-via-spatial shifts account). They suggested a greater reliance on a heuristic where information from the visuo-spatial system is fed into the decision when attentional load is high. Indeed, McCrink and Hubbard (2017) observed a stronger OME in non-symbolic addition and subtraction problems in a dual-task situation where participants divided attention between numerosity processing and a secondary feature-detection task compared to a single-task context where only the non-symbolic arithmetic problems were solved (McCrink & Hubbard, 2017).

Heuristics have also been proposed to account for SAAs in arithmetic cueing paradigms. Specifically, associations between operators and space might stem from conceptual metaphors associating operations and space, which might help subsequent calculation by providing heuristics narrowing down the range of possible answers (Andres et al., 2020). For example, by associating addition and subtraction to the right and left side of space (respectively), participants might come to infer that "more is right" and "less is left." They will thus shift their attention either to the right or to the left to anticipate that the result of an addition will be larger than the first operand, whereas the result of a subtraction will be smaller than the first operand.

Although heuristics such as those described above can explain a range of findings, they should not only apply to addition and subtraction, but also to multiplication and division. First, because multiplication leads to outcomes generally larger than the first operand and division leads to outcomes generally smaller than the first operand, participants should overestimate results of multiplication and underestimate results of division. In line with this prediction, Katz and Knops (2014) did observe overestimations in multiplications and underestimation in division. However, this was limited to the non-symbolic notation. No OME was observed for symbolic multiplication or division. This pattern remained stable even when approximate calculation (as compared to exact retrieval from rote memory) was endorsed by presenting only incorrect response choices for symbolic problems amongst which the one closest to the correct outcome should be selected (Katz et al., 2017). In the non-symbolic multiplication and division problems, the OME correlated with the reorienting cost due to invalid cueing in a Posner task. Therefore, while the presence of the OME in non-symbolic multiplication and division is consistent with the heuristics approach, the absence of the effect in symbolic notation and the correlation with the reorienting effect are not predicted by this account. Second, in arithmetic cueing tasks, a heuristic such as "more is right" should apply to multiplication as much as it applies to addition. Yet, multiplication has not been found to be associated with a rightward shift of attention (Mathieu et al., 2016). The multiplication operator ('×') has also been found to elicit less activity than the addition operator ('+') in brain regions underlying spatial attention (Mathieu, Epinat-Duclos, Léone, et al., 2018a, Mathieu, Epinat-Duclos, Sigovan, et al., 2018b). Therefore, studies on multiplication and division have generally failed to support the heuristic account of SAAs. The observation that SAAs are flexibly adapting to contextual factors such as the right-to-left orientation of an external response medium further undermines the heuristics account. That is, finding that addition can induce biases to the left and subtraction to the right side of space (Klein et al., 2014; Pinhas et al., 2015) is at odds with the heuristics account. One might argue that the heuristic operates on the situated, context-dependent representation of mental magnitude, which would bias participants' responses to the left (towards larger numbers) for addition and to the right (towards smaller numbers) for subtraction in the study by Klein et al. (2014). Yet, even under this interpretation, we argue that it is unclear why the heuristic would bias only the second, corrective saccade rather than the first saccadic landing point, which supposedly reflects the result obtained via heuristic problem solving.

Spatial competition and arithmetic heuristics and biases (AHAB) account

The spatial competition account assumes that SAAs (including the OME) result from the competing spatial biases invoked by the operands, the operation sign, and the result of an arithmetic problem. This account has been recently expanded and replaced by the more general idea (termed arithmetic heuristics and biases account or AHAB account) that different biases interact during mental arithmetic, namely the anchoring bias, the operator-space association, and the more-or-less heuristic (Mioni et al., 2021). For example, the anchoring bias predicts that for problems with matched outcome, subtraction would induce an overestimation compared to addition because of the comparably larger first operand ($9 - 3 = 6$ vs. $4 + 2 = 6$). The operator-space association predicts a rightward bias for additions and a leftward bias for subtractions (though to take effect, this association depends on the use of spatially distributed responses). Finally, the more-or-less heuristic results from the repeated experience that addition leads to larger outcomes and subtraction to smaller outcomes. The AHAB framework therefore integrates elements from the previously described accounts.

The AHAB account is supported by a number of findings. For example, there is evidence that SAAs are not uniquely observed after the second operand or during calculation per se. Several studies have found that arithmetic operators may be associated with shifts of attention before the second operand is even known to participants (Hartmann et al., 2015; Liu, Cai, et al., 2017a; Salvaggio, Masson, et al., 2022b), though these shifts may only occur when operators are preceded by an operand (Liu, Cai et al., 2017a; Pinhas et al., 2014). The first indication that arithmetic operators do have spatial association comes from a study by Pinhas et al. (2014), who asked participants to classify arithmetic operators ('+' or '-') using different response mappings (either the left or right hand). The study showed that '+' signs were classified faster with the right than the left hand, whereas '-' signs were classified faster with the left than the right hand (see also Brennan et al., 2021) for a replication of that finding). Neuroimaging findings also indicate a relation between arithmetic operators and spatial attention. For example, both Mathieu, Epinat-Duclos, Léone, et al. (2018a) and Mathieu, Epinat-Duclos, Sigovan et al. (2018b) measured brain activity of children and adults while they were presented with a '+' sign in anticipation of a forthcoming addition problem. Interestingly, the mere presentation of that '+' sign elicited enhanced activity in brain regions that were identified in the same experiments as supporting saccadic eye movements. These findings thus suggest that '+' signs are processed in brain regions that underlie spatial attention, in keeping with behavioral findings showing that such operators do elicit shits of attention (to the right side of space).

Despite these findings supporting the operator-space association, a number of challenges remain for the AHAB account. For example, no study with two-operand problems has provided empirical support for the anchoring bias. The AHAB account also provides some very specific predictions that have not been confirmed yet. For example, it assumes that "when the sign–space association is largely irrelevant to the task, [...] the anchoring bias outweighs the more-or-less heuristic" (p. 538; Mioni et al., 2021), leading to inverse OMEs. This is, however, at odds with results from studies that used no arithmetic operator (McCrink et al., 2007) and show regular OMEs even with matched results (Knops, Viarouge et al., 2009b). Overall, the boundary conditions of the interaction between these biases remain to be specified and – importantly – empirically tested. Considering the difficulties associated with mapping paradigms (see above), this test should make use of paradigms that do not require the participants to map an internally generated numerical outcome onto an external non-numerical dimension such as line length or temporal duration. This additionally required mapping may in and by itself induce biases that obfuscate the exploration of the factors underlying SAAs.

Evaluating the theoretical accounts against the observed OME and arithmetic cueing effects

Overall, all theoretical accounts can successfully explain a number of findings. At the same time, they also face empirical challenges that require the precise definition of boundary conditions. This conclusion is substantiated by some of the results that have been revealed in our meta-analyses of OME and arithmetic cueing studies and that were not evident when assessing the literature qualitatively. We briefly summarize these findings before presenting a tentative theoretical framework that accommodates them. The new framework makes it possible (a) to explain the observed dissociations between OME and attentional curing studies, and – more importantly – (b) to derive testable predictions that may inspire future studies.

A key finding from our meta-analyses is that, when addition is compared to subtraction, both an OME and a lateralization effect after arithmetic cueing are consistently observed across studies. The effect is in the small to medium range in arithmetic cueing studies ($d = 0.37$) and in the medium to very large range ($d = 0.96$) in OME studies. Combined with findings from studies tracking eye and hand movements during arithmetic calculation, the current literature clearly supports the view that arithmetic processing is subject to biases that indicate (in the case of tracking and arithmetic cueing studies) or suggest (in the case of OME studies) spatial processing. While the spatial interpretation of arithmetic cueing effects is obvious due to the explicit interaction between the spatial and the numerical dimensions in these paradigms, the OME is only a numerical bias. Under the premise of a spatial organization of numerical magnitude, it is nevertheless suggesting the involvement of spatial processes during mental arithmetic (Knops, Thirion et al., 2009a). The only major exception to this picture comes from studies on the OME that require participants to transcode the results to an external spatial scale (position on a line or line length). At least for the moment, these do not provide coherent evidence for an OME. While a small to medium pooled effect size emerged for zero problems, the effect size for problems involving operands that are different from zero was equivalent to zero. As stated before, we argue that this paradigm involves an additional mapping of an internal representation onto an external spatial dimension that is far from trivial and open to a number of different strategies. Disentangling how these strategies might influence performance remains an interesting challenge for future work.

Despite the fact that most OME and arithmetic cueing studies reliably find response biases, the operation driving the effect appears to differ between OME and arithmetic cueing studies. On the one hand, the OME is driven by an underestimation of the result in subtraction ($d = -1.38$) rather than by an overestimation in addition ($d = -0.09$). On the other hand, the lateralization effect after arithmetic cueing is driven by a rightward bias for addition ($d = 0.34$) rather than a leftward bias for subtraction ($d = -0.01$). In other words, arithmetic cueing effects dissociate from the OME since the former are mainly driven by an association between addition and the right side of space while the latter is mainly driven by an underestimation in subtraction problems. In fact, this dissociation is in line with a recent study that measured attentional focus via a target detection task in the context of non-symbolic arithmetic (Glaser & Knops, 2023). The authors did not observe any arithmetic cueing effects, while at the same time replicating the OME that was driven by subtraction only (Glaser & Knops, 2023). None of the theoretical frameworks described above can comprehensively explain such a dissociation, which calls for refined theorization. Clearly, such a dissociation is relatively problematic for theoretical accounts that provide a joint framework for both effects, such as the attentional shift account or the heuristics account.

We can see at least two potential explanations for the fact that the OME and the arithmetic cueing effects dissociate. First, it might be that either the OME or the arithmetic cueing effect (or both) does not reflect attentional shifts along the MNL but stems from other (and different) sources, as suggested by some studies. For instance, the OME has been explained by non-attentional accounts, such as the compression account (Chen & Verguts, 2012; McCrink et al., 2007). It has also been proposed that arithmetic cueing effects may be due to heuristics associating operations with space (McCrink et al., 2007; McCrink & Wynn, 2009). Yet, it is unclear how these effect-specific accounts might explain that (a) the OME would be driven by subtraction rather than addition and (b) arithmetic cueing would be stronger for addition than subtraction. A specific concern with the compression account is also that it does not provide an explanation for the resemblance of parietal activation patterns associated with attentional shifts and arithmetic operations (Knops, Thirion et al., 2009a, Knops, Viarouge et al., 2009b).

Second, it is possible that the dissociation between the OME and the arithmetic cueing effect might be more apparent than real. That is, both effects could still stem from attentional shifts along the MNL, but confounding experimental factors might allow for different biases to intervene and obscure the effects. One factor that is – albeit not perfectly – confounded with this distinction is the format of the problems. Specifically, studies that measure the OME have mostly used non-symbolic stimuli while arithmetic cueing studies mostly utilized symbolic stimuli. Critically, the choice of stimulus format (as well as type of arithmetic problem) can favor different factors to influence the arithmetic processing.

For example, the use of non-symbolic numerosities in most studies examining the OME provides an opportunity for visual-perceptual biases (which are not involved in processing symbolic stimuli) to interfere with arithmetic processing (Santens et al., 2010). Candidate biases include recently described attractive serial dependency effects (Fornaciai & Park, 2020). The core idea is that the numerosities presented as operands leave a memory trace that influences the processing of subsequently presented items. The first operand may leave activation traces which serves as an attractor for subsequently presented numerosities (operand attractor hypothesis). In subtraction problems where the first operand is always larger than the second operand this would lead to an overestimation of the second operand, which in turn would lead to an underestimation of the outcome. In addition problems, the situation is less clear since the first operand is not necessarily larger than the second operand and hence sequential attraction may go either way – diminishing potential biases. Therefore, operand attractor may be a factor enhancing the underestimation of subtraction. Note that serial attraction effects may also be observed in the context of symbolic arithmetic, where they are sometimes referred to as "anchoring" effects. Although such effects may also affect the perceived numerical magnitude of symbolic numbers (Charras et al., 2012; Pinhas & Fischer, 2008), due to the exact nature of the verbal labels, we would argue that these effects are smaller for symbolic stimuli.

In addition to serial attraction between operands, there might also be an overall tendency to underestimate sets of items in non-symbolic numerosities. This would offset all final estimates in non-symbolic tasks to the left of the MNL and enhance even further the OME observed with subtraction. While we see that this theoretical stance is not unproblematic since the underestimation mostly affects transcoding to verbal formats, which is not required systematically, we propose that participants routinely apply verbal labels to the non-symbolic quantities. Nonetheless, an open question is how much this applies to paradigms that do not require any transcoding at all.

Finally, strategy choice is also a factor that may affect symbolic arithmetic to a greater extent than non-symbolic arithmetic. Two prominent strategies that may impact the manifestation of attentional biases are direct retrieval of solutions from long-term memory for multiplication problems and the solution of subtraction problems via addition (e.g., 8 + ? = 12 for 12 – 8 = ?; Campbell, 2008; Torbeyns et al., 2018). Direct retrieval from long-term memory would leave little room for any attentional bias compared to an estimation procedure for non-symbolic multiplication problems, which is consistent with the findings from Katz and Knops (2014). Solving subtraction problems via an addition strategy, in turn, would explain smaller biases in symbolic subtraction problems.

In sum, our meta-analysis showed that arithmetic cueing effects are mainly driven by addition problems while the OME is mainly driven by subtraction problems. This may coincide with the idea that arithmetic cueing paradigms are particularly strategy-sensitive (e.g., subtraction-by-addition might reduce spatial associations for subtraction problems) while OME paradigms are more sensitive to biases induced

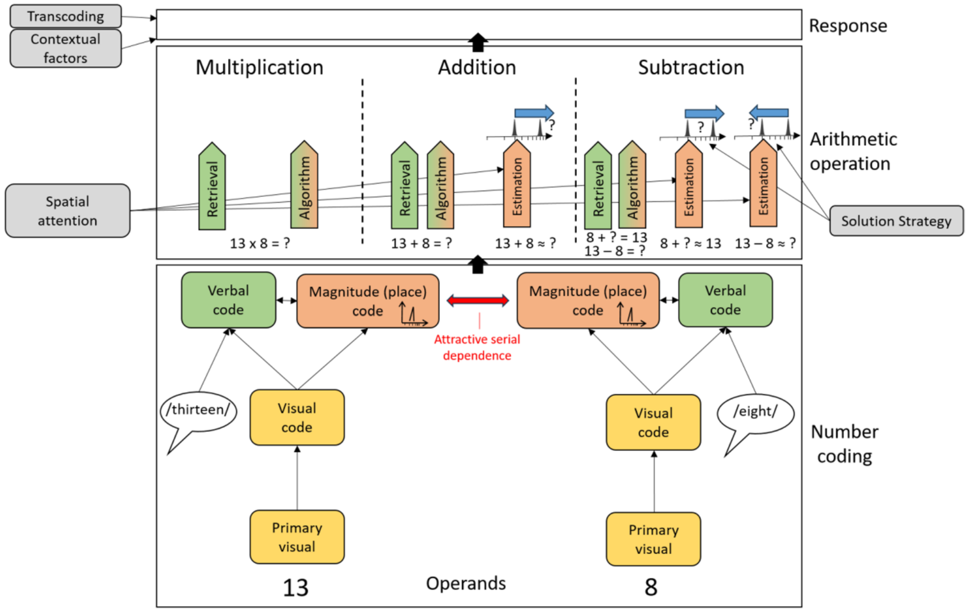

Fig. 8 Adaptive Pathways in Mental Arithmetic (APiMA) framework for symbolic numbers (see text for details). The figure shows how the model considers the manipulation of the numbers '13' and '8' within multiplication, addition, and subtraction

by serial dependency effects that may be more prominent in the non-symbolic notation. Note that we do not mean to imply here that no other factors may influence the size or presence of arithmetic cueing and OM effects. For instance, the effects may be affected by the range of numbers (e.g., single- vs. multi-digit), the difficulty of the problems (e.g., small vs. large, problems involving carrying or borrowing versus problems that do not involve these), or even some characteristics of the problems that may influence spatial associations independently of the operation (e.g., whether subtraction and addition are matched for operands or results),[2] as suggested by several studies (e.g., Salvaggio, Andres et al., 2022a, Salvaggio, Masson et al., 2022b; Masson & Pesenti, 2023).

The adaptive pathways in mental arithmetic (APiMA) framework

Though a number of factors may influence arithmetic cueing and OM effects (see above), one prominent factor may be a difference in notation format (non-symbolic vs. symbolic) between most studies investigating the OME and most studies investigating arithmetic cueing. This might potentially explain why the effects are driven by different operations (subtraction for the OME and addition for arithmetic cueing). To illustrate this point, we introduce the adaptive pathways in mental arithmetic framework (APiMA; Figs. 8 and 9), which summarizes processing instances during mental arithmetic as well as the underlying codes with their most prominent characteristics. The APiMA model incorporates basic notions of the Triple Code Model (Dehaene & Cohen, 1995), the separated input pathways stipulated by Santens et al. (2010), and a parallel pathway assumption of mental arithmetic, hypothesizing that approximate estimation and verbally mediated calculation strategies are carried out in parallel (Ashcraft & Stazyk, 1981). The APiMA focuses on perceptual and semantic elaboration processes as opposed to retrieval of arithmetic facts from long-term memory, although these processes need to operate in synchrony (Klein & Knops, 2023).

The APiMA model provides a detailed overview of instances where notation-specific biases may operate. This is because the processing pathways for symbolic and non-symbolic information differ and these differences run through all processing steps from perception over semantic elaboration until response-related instances. When numbers are presented symbolically (either through visual or auditory stimulation; see Fig. 8), quantity may be represented using both a verbal and a magnitude code (Dehaene & Cohen, 1995). As hypothesized by Dehaene & Cohen (1995), these codes provide the basis for giving an exact answer to the

[2] Because results of addition problems are larger than results of subtraction problems when problems are matched for operands, it is difficult to disentangle spatial associations that would be due to the type of operation from associations that would be driven by the size of the result. A potential way to disentangle these factors is to match addition and subtraction problems in terms of results rather than operands (e.g., Knops, Viarouge & Dehaene, 2009b; Masson & Pesenti, 2014).

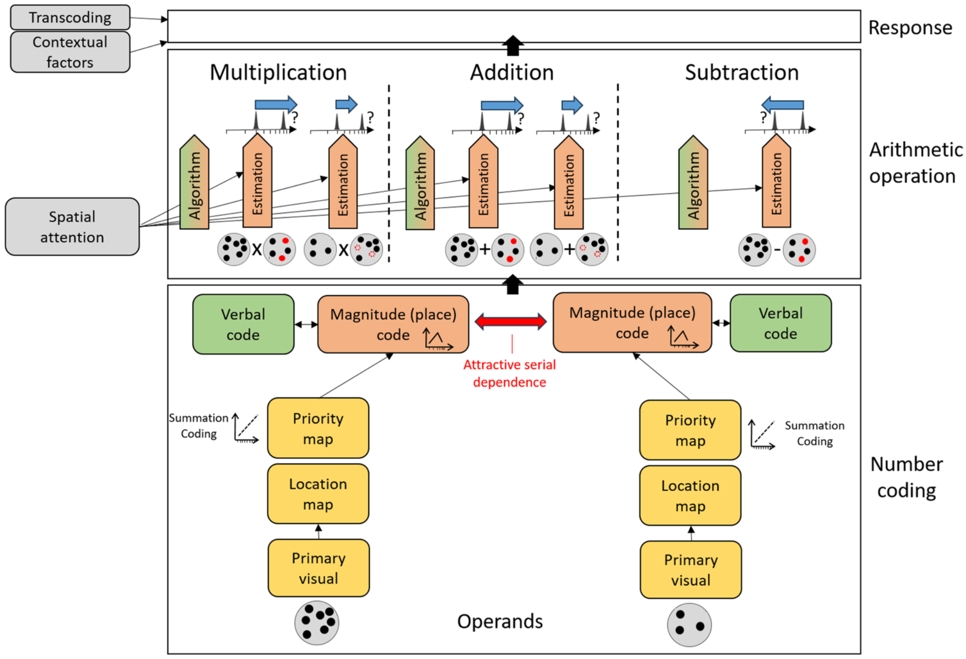

Fig. 9 Adaptive Pathways in Mental Arithmetic (APiMA) framework for non-symbolic numerosities (see text for details). The figure shows how the model considers the manipulation of seven dots and three dots within multiplication, addition, and subtraction. Full and dotted red dots on the upper panel represents hypothetical changes in numerosity representations due to attractive serial dependence. For addition and multiplication, attractive serial dependence tends to increase the number of dots of the second operand when the largest numerosity is the first operand (leading to overestimation) while it tends to decrease the number of dots of the second operand when the smallest numerosity is the first operand (leading to underestimation). For subtraction, attractive serial dependence systematically tends to increase the number of dots of the second operand because the largest numerosity is always the first operand (leading to underestimation)

arithmetic problems using either verbal retrieval or algorithmic computing. However, the APiMA model also assumes that the magnitude code may also provide an estimation of the result, through spatial shifts along the MNL.[3] These may be useful to narrow down the range of possible answers (Salvaggio, Masson et al., 2022b). Critically, because symbolic multiplication problems are learned by rote in school, it is largely assumed that these are directly retrieved from memory in adults (or solved through backup strategies if retrieval is not possible). As such, studies have failed to find arithmetic cueing and OM effects with symbolic multiplication problems (Katz & Knops, 2014; Mathieu et al., 2016). But an estimation of the result might be relatively frequent when adding numbers (leading to a rightward shift attention along the MNL), at least more so than when multiplying numbers[3]. Much like addition, the APiMA model also assumes that estimation is present in subtraction as well. However, because subtraction problems can be solved either by backward counting or subtraction-by-addition (Campbell, 2008), shifts along the MNL may occur either leftward or rightward depending on the strategy. Overall, the APiMA model explains why arithmetic cueing effects are observed more reliably in symbolic addition than in symbolic subtraction or multiplication.

Though number coding pathways differ between symbolic and non-symbolic quantities, non-symbolic numerosities may also be represented using a verbal and a magnitude code (see Fig. 9). These may also provide the basis for giving an exact answer to the arithmetic problems using algorithmic computing (verbal retrieval being much less prevalent with non-symbolic stimuli). As for symbolic numbers, the APiMA model also assumes that the magnitude code may provide an estimation of the result through spatial shifts along the MNL. However, as detailed above, there might be serial dependency between magnitude representations of two sequential numerosities, which would lead to either an overestimation of the second operand when the first operand is the largest numerosity or an underestimation of the second operand when the first operand is the smallest numerosity (see red dots on sample numerosities in

[3] Note that it has been proposed that shifts along the MNL may also provide the exact answer to symbolic arithmetic problems in some situations, as these shifts could correspond to counting procedures that have been automatized (Poletti et al., 2023; Uittenhove et al., 2016). This process, however, is believed to be restricted to operands that are smaller than 4 and therefore cannot account for the range of associations between symbolic arithmetic and space (though it might account for some associations in small problems; Mathieu et al., 2016).

Fig. 9). In subtraction, the first operand is always the largest as non-symbolic subtraction cannot typically be associated with negative results. This would lead to an overestimation of the second operand and an enhancement of the leftward shift along the MNL. In addition, because the first operand may be the largest or the smallest, the second operand may be either overestimated or underestimated. Rightward shifts along the MNL may therefore be either enhanced or diminished, and on average weaker in addition than in subtraction. The model predicts that multiplication should be similar to addition in that respect. Nonetheless, the APiMA framework accounts for the observation that OME has been observed with non-symbolic but not with symbolic multiplication and division because the latter predominantly calls on the recall of arithmetic facts from long-term memory who have a weak association with the semantic code only (Didino et al., 2015; Katz & Knops, 2014).

The APiMA model is based on the assumption that attentional shifts underlie both the OME and the arithmetic cueing effects. It further assumes that the consistency of attentional shifts with the overall displacement along the spatial numerical representation leads to a stronger bias. As a second mechanism, APiMA includes the notion of serial attraction effects that affect the perceived magnitude of the operands (and potentially the response alternatives). Interestingly, serial attraction effects may modulate attentional biases in predictable ways.

For addition, we can differentiate between problems where (a) the first operand (O1) is smaller than the second (O2) and the result (R), or (b) problems where the O2 is smaller than O1. According to the consistency hypothesis, both problem types lead to an OME. If we additionally assume serial attraction effects, O1 influences ("attracts") the subjectively perceived numerical magnitude of O2. In problems of type (a), this leads to a smaller subjective magnitude of O2 compared to problems of type (b), all else being equal. Consequently, this would lead to a larger OME for problems of type (b) where the O2 is smaller than O1. Interestingly, this is what Charras and colleagues observed in a series of experiments (Charras et al., 2012, 2014) where the order of operands in addition problems was systematically varied. They observed a larger overestimation for problems with operands in descending order (e.g., 26 + 22) compared to problems with the inverse operand order (i.e., 22 + 26).

For subtraction problems, too, we can differentiate two types of problems. In problems of type (a), O2 is smaller than O1 but larger than the result (e.g., 24 – 15 = 9). In problems of type (b), O2 is smaller than both O1 and the result (e.g., 24 – 9 = 15). According to the consistency hypothesis, problems of type (a) should produce a larger OME due to the consistent displacement to the left compared to problems of type (b). At the same time, problems of type (a) are more prone to the application of a subtraction-via-addition strategy, which should diminish the OME. Hence, the analysis of the adopted strategy appears a necessary factor in the future exploration of attentional biases in the context of mental arithmetic. Sequential attraction effects, however, would lead to a larger OME in problems of type (b) compared to problems of type (a).

We believe that the above hypotheses, which can be inferred from the APiMA framework, represent exciting starting points for a further refinement of the theoretical mechanisms underlying OME and arithmetic cueing effects in the context of mental arithmetic.

Conclusion

More than 15 years has passed since Hubbard et al. (2005) hypothesized that mental arithmetic involves attentional movements along the MNL. As reviewed here, there is now convergent evidence that arithmetic calculation is indeed associated with response biases that appear to be spatial in nature. Although there is still a debate about whether these biases reflect movements along a MNL per se, studies indicate that such spatial associations are not simply a byproduct of calculation (Dormal et al., 2014; Masson & Pesenti, 2016; Masson, Pesenti, & Dormal, 2017b). Rather, they might reflect mechanisms that are at the heart of arithmetic processing and even pertain to the arithmetic combination of non-numerical (i.e., temporal) quantities (Bonato et al., 2021). That being said, the literature also raises a number of challenges for future theories and paradigms. First, although the OME and the arithmetic cueing effect are often seen as two manifestations of the same phenomenon, some may doubt that they stem from the same mechanism. Second, the framework emerged from analyzing studies that examined arithmetic-space associations in a horizontal plane (i.e., left-right). Future frameworks might embrace number-space interactions in down-up or near-far planes (Holmes, 2012; Hartmann et al., 2014; Aleotti et al., 2020). The literature on associations of arithmetic with these alternative dimensions, however, is still scarce at the moment (e.g., Wiemers et al., 2014), mostly exploiting eye movement recordings (Blini et al., 2019; Hartmann, 2022). Finally, the studies analyzed here are characterized by some degree of heterogeneity in terms of tasks and materials, which makes it difficult to evaluate to what extent spatial biases in mental arithmetic depend on specific task features (e.g., non-symbolic vs. symbolic quantities, problem size, response output). The proposed framework may guide future work that seeks to elucidate the cognitive characteristics of the described spatial-numerical associations. For example, the model we propose assumes that the dissociation observed between OME and arithmetic cueing studies has more to do with a difference in the nature of stimuli (non-symbolic vs. symbolic) than in underlying mechanisms. Specifically, a greater variety in strategies used to solve symbolic subtractions may explain why arithmetic cueing effects are stronger in addition than subtraction. Critically, the model predicts that problems solved by subtraction-by-addition should be associated with a rightward shift while other problems should be associated with a leftward shift. We also predict that attractive serial dependence between non-symbolic numerosities and a tendency to underestimate may explain why the OME is stronger in subtraction than addition. Here, the model notably predicts that the OME observed in addition problems should be stronger when the first operand is larger than the second (compared to the other way around). These are testable predictions that future studies may investigate.

On a final note, the current review exclusively focuses on adult participants. Only a small number of studies have investigated the development of spatial biases during mental arithmetic in children (Díaz-Barriga Yáñez et al., 2020; Masson et al., 2024; Pinheiro-Chagas et al., 2018). Yet, we believe that this research is crucial as it might inform on the mechanisms through which these biases emerge and how they are modulated by instructional context, thereby shedding light on the sources of both the OME and arithmetic cueing effects in expert adults. On a more general note, our findings reverberate with recent efforts to characterize the relation between internal and external attention that have been theorized to operate via shared neural and cognitive mechanisms (Kiyonaga & Egner, 2013). The current results support this idea by demonstrating that attentionally mediated arithmetic operations on an internal representational space affect the perceptual performance of external visual stimuli and vice versa. Whether or not the reciprocal influence is entirely symmetric or not remains to be seen in future studies (Lim & Pratt, 2023).

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Data Availability The data and full list of identified papers for the meta-analyses are available at https://osf.io/5gcd3/?view_only=144d8aab62884d608d3762f9b0bdd06d.

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