CHAPTER

2

The Interplay Between Learning Arithmetic and Learning to Read: Insights From Developmental Cognitive Neuroscience

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INTRODUCTION

Over the past two decades or so, studies from both psychology and cognitive neuroscience have converged on the view that mathematical and linguistic abilities are largely separated. For example, behavioral studies have found that preverbal infants (Cordes & Brannon, 2008), as well as individuals with very limited mathematical language (Pica, Lemer, Izard, & Dehaene, 2004), can intuitively process numerical quantities when these are presented in a nonsymbolic format (e.g., dot patterns). Because these intuitions appear to be related to the acquisition of formal mathematical skills later on (Feigenson, Libertus, & Halberda, 2013), it is increasingly believed that these core numerical skills—rather than linguistic skills—provide the foundation for the emergence of abstract mathematical concepts (Dehaene & Cohen, 2007). In line with this idea, neuroimaging studies in adults indicate that many numerical tasks consistently recruit a brain system that is largely dissociated from regions involved in linguistic computations (Nieder & Dehaene, 2009). This system, notably, includes the intraparietal sulcus (IPS), a region that is believed to house a representation of numerical quantities (Nieder & Dehaene, 2009).

Therefore, one would think that the acquisition of mathematical skills in children should be fairly independent from the acquisition of linguistic skills. Yet, this does not appear to be the case, at least as far as two of the most foundational mathematical and linguistic skills are concerned: learning arithmetic and learning to read. Indeed, two relatively independent lines of evidence argue in favor of a relationship between the acquisition of these skills. The first line of evidence comes from correlational studies. These studies show that there is a correlation between arithmetic and reading abilities across children (Durand, Hulme, Larkin, & Snowling, 2005; Hart, Petrill, Thompson, & Plomin, 2009; Hecht, Torgesen, Wagner, & Rashotte, 2001). For example, in a study conducted on 162 children from 7 to 10 years of age, correlations between arithmetic skills and single-word reading abilities ranged from .50 to .60 (Durand et al., 2005). Conversely, studies have also found that mathematical abilities in children can predict later reading outcomes, sometimes even better than early reading skills (Duncan et al., 2007; Lerkkanen, Rasku-Puttonen, Aunola, & Nurmi, 2005).

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Therefore, correlational studies point to a relatively clear relationship between the acquisition of arithmetic and reading skills in children.

The second line of evidence that suggests a relationship between learning math and learning to read comes from the study of learning difficulties. Some children can show persistent difficulties acquiring basic arithmetic skills, even though they may have average intelligence and benefit from adequate schooling (Kaufmann & von Aster, 2012). This condition, called developmental dyscalculia or math learning disability, affects from 5% to 10% of children worldwide (prevalence varies depending on diagnostic criteria) (Kaufmann & von Aster, 2012). Although such prevalence rates are similar to those observed with children who show persistent reading difficulties (i.e., developmental dyslexia), dyscalculia is much less researched than dyslexia. Yet, both disabilities may be related because estimates of the prevalence of one disability given the other (i.e., comorbidity rate) are estimated at around 40% (Wilson et al., 2015). Therefore, many children who show persistent struggles with learning math also show persistent struggles with learning to read.

Overall, both (1) the correlation between arithmetic and reading scores and (2) the comorbidity between dyscalculia and dyslexia suggest some overlap between the acquisition of arithmetic and reading skills in children. At first glance, this appears to somewhat conflict with the aforementioned evidence that the neural mechanisms supporting mathematical skills in adults are largely independent from those supporting linguistic skills. Against this background, the goal of this chapter is to review some recent developmental cognitive neuroscience findings that are relevant for understanding the observed relationship between the acquisition of arithmetic and reading skills in children. First, we will evaluate whether developmental neuroimaging studies support what is perhaps the most popular explanation of the link between arithmetic and reading skills: the idea that arithmetic learning in children involves phonological processing mechanisms also involved in reading (Ashkenazi, Black, Abrams, Hoeft, & Menon, 2013). Second, we will review the more recent proposal that the relationship between arithmetic and reading skills may also be explained by the fact that both rely on procedural memory and more specifically on the ability to automatize procedural knowledge.

ARITHMETIC LEARNING AND VERBAL-PHONOLOGICAL PROCESSING

Acquiring the ability to identify and manipulate the sound structure (i.e., the phonology) of a word is a critical component of learning to read. Early phonological processing skills are strong predictors of later reading

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performance in typically developing children (Melby-Lervåg, Lyster, & Hulme, 2012) and are impaired in dyslexia (Vellutino, Fletcher, Snowling, & Scanlon, 2004). The neural mechanisms supporting phonological processing have been thoroughly researched over the past decades. Taken together, neuroimaging studies indicate that these mechanisms are located in regions of the left temporoparietal cortex, including the left superior and middle temporal gyri (STG and MTG, respectively) and the left angular gyrus (AG) (Vigneau et al., 2006) (see Fig. 2.1). Studies further indicate that reading acquisition is associated with developmental decreases of activity in left temporoparietal regions (Martin, Schurz, Kronbichler, & Richlan, 2015). This suggests that children are more likely than adults to rely on phonology-based reading. Critically, it has often been argued that such verbal-phonological mechanisms in the left temporoparietal cortex might also contribute to arithmetic learning (Prado, Mutreja, & Booth, 2014; Zamarian, Ischebeck, & Delazer, 2009). This idea mainly comes from the triple-code model (Dehaene & Cohen, 1995), a popular neurocognitive model of numerical processing, which is briefly described in the following.

The Triple-Code Model

One of the motivations for the triple-code model proposed by Dehaene and Cohen (1995) was to account for task-specific patterns of

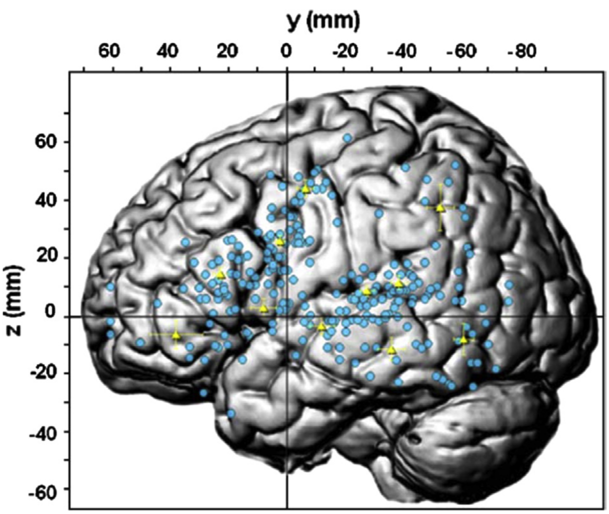

FIGURE 2.1 Location of activation peaks in fMRI studies on phonological processing. Peaks are plotted in blue on a 3D rendering of a left hemisphere. *Reproduced from Vigneau, M., Beaucousin, V., Herve, P. Y., Duffau, H., Crivello, F., Houde, O., et al. (2006). Meta-analyzing left hemisphere language areas: Phonology, semantics, and sentence processing.* Neuroimage, 30*(4), 1414–1432.*

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mathematical impairments (as well as dissociated patterns of impaired performance in mathematical tasks) that are often observed in brain-damaged patients (Cohen & Dehaene, 2000; Lemer, Dehaene, Spelke, & Cohen, 2003; van Harskamp, Rudge, & Cipolotti, 2002, 2005). Dehaene and Cohen hypothesized that numbers may be represented according to three different codes in the adult brain: a visual-Arabic code that would support the identification of visually presented (strings of) digits, a magnitude code that would support semantic knowledge about numerical quantities, and a verbal code that would support the representation of numbers as sequences of written or spoken words. Because one code could be impaired while the others would be spared, the triple-code model constitutes a powerful framework for explaining dissociations in patients (Dehaene, Molko, Cohen, & Wilson, 2004). Of particular interest here is the verbal code, which is posited to be important when retrieving answers of well-known arithmetic facts from memory. For instance, Dehaene and Cohen argue that "arithmetic facts such as $2 \times 3 = 6$ cannot be retrieved unless the problem is coded into a verbal code 'two times three…' which then triggers the retrieval of the result 'six' in the same verbal format" (Dehaene & Cohen, 1995, p. 87). Thus, although there are bidirectional links between codes allowing for indirect routes, the model proposes that arithmetic problems whose answers are well known are not associated with any particular semantic access to underlying quantities. Rather, these problems might be solved by directly retrieving the associated answer from declarative memory. This may of course be case of problems that are explicitly learned by rote in school, such as single-digit multiplication problems. But it should be noted that the verbal-phonological code is also thought to be used for problems that are not necessarily explicitly learned by rote but are particularly well practiced in school (and solved with no apparent difficulty by educated adults). This is, for instance, the case of single-digit addition problems (e.g., $2 + 3 = 5$). Therefore, the triple-code model suggests that some arithmetic skills might rely on the brain mechanisms underlying verbal-phonological processing in the left temporoparietal cortex (as well as the basal ganglia) (Dehaene, Piazza, Pinel, & Cohen, 2003).

Evidence From Studies in Adults

Looking over two decades of neuroimaging research, there seems to be converging support for the triple-code model's assumption that left temporoparietal areas are involved in processing well-learned arithmetic facts in adults. For example, studies that have contrasted the solving of multiplication problems to various control tasks (e.g., number comparison, letter or digit matching, number storage in working memory) have consistently found multiplication-specific activity in regions of the

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left temporoparietal cortex, including the left AG (Chochon, Cohen, van de Moortele, & Dehaene, 1999; Fulbright, Manson, Skudlarski, Lacadie, & Gore, 2003; Gruber, Indefrey, Steinmetz, & Kleinschmidt, 2001; Jost, Khader, Burke, Bien, & Rosler, 2009). Other studies have found that multiplication is also associated with greater activity than subtraction (i.e., an operation that is thought to rely to a lesser extent on rote memorization than multiplication, Campbell & Xue, 2001) in the left AG (Lee, 2000) and left STG/MTG (Andres, Michaux, & Pesenti, 2012; Andres, Pelgrims, Michaux, Olivier, & Pesenti, 2011; Prado et al., 2011). Activity in the left MTG and the left AG has also been shown to increase as fluency with multiplication problems increases (Ischebeck, Zamarian, Egger, Schocke, & Delazer, 2007). Finally, the left MTG has been found to be more responsive to single-digit multiplication than to single-digit addition problems (Zhou et al., 2007), perhaps because single-digit multiplication problems are even more likely to be directly retrieved from memory than single-digit addition problems (Campbell & Xue, 2001). Finally, the involvement of the left temporoparietal cortex in processing well-known arithmetic facts is also supported by studies that (1) explicitly linked brain activity to self-report of strategies (problems reported to be retrieved vs. calculated) (Grabner et al., 2009) and (2) compared simple with more complex arithmetic problem solving (Grabner et al., 2007; Stanescu-Cosson et al., 2000). Overall, then, there is converging support for the idea that well-known arithmetic facts activate regions of the left temporoparietal cortex in adults.

However, there is at least one important issue with the aforementioned studies: none of these studies have independently localized the brain mechanisms supporting verbal-phonological processing in the left temporoparietal cortex. This issue is particularly problematic with the AG, which has consistently been found to be a component of the default mode network (DMN) (Raichle, 2015; Seghier, 2013). The DMN is a network of regions that contribute to internal modes of cognition and are typically less *deactivated* (with respect to some low-level baseline) when a condition is not attention demanding (and therefore relatively easy) than when it is attention demanding (and therefore more difficult) (Raichle, 2015). It is thus worrisome that most of the studies that have identified the left AG in arithmetic tasks precisely report less *deactivation* in this region for contrasts that systematically involve a comparison between a relatively easy and a relatively difficult condition. This is, for example, almost systematically the case when comparing simple with complex problems (Grabner et al., 2007; Stanescu-Cosson et al., 2000), well-known multiplication problems with lesser-known subtraction problems (Lee, 2000), trained problems with untrained problems (Ischebeck et al., 2007), or problems reported to be retrieved from memory with problems reported to be calculated (Grabner et al., 2009). In other words, differences in levels of deactivation in the left AG

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during arithmetic problem solving are often correlated with differences in behavioral performance. Therefore, the activation of the left temporoparietal cortex in arithmetic tasks may be an artifact of a difference in difficulty between conditions.[1]

To directly investigate whether phonological processing mechanisms of the left temporoparietal cortex specifically contribute to arithmetic problem solving, Prado et al. (2011) asked the same group of adult participants to perform both a word-rhyming task and an arithmetic task in an MRI scanner. The rhyming task was associated with activity in the left MTG. Brain activity was then measured in this specific cluster when participants were asked to evaluate the validity of either single-digit multiplication or subtraction problems. Not only was the MTG cluster *activated* in the multiplication condition (rather than *deactivated*) but also activity was significantly higher in the multiplication than in the subtraction condition. Thus, processing multiplication facts does involve a brain region involved in verbal-phonological processing in adults and more so than processing subtraction problems. This may be because unlike subtraction problems, multiplication problems are learned by rote in school and more likely to systematically be associated with fact retrieval rather than numerical manipulation (Campbell & Xue, 2001).

It is important to consider, however, that evidence coming from adult studies mainly provides an indirect understanding of the brain systems that underlie simple arithmetic learning. First, simple arithmetic is learned in elementary school and adults can be considered experts in those tasks. Therefore, such studies may not inform on the mechanisms enabling learning per se. Second, although training studies in adults (e.g., Bloechle et al., 2016; Ischebeck et al., 2007) can provide valuable information regarding such learning mechanisms, it is unclear to what extent the neural changes observed in a mature brain mimic the neural changes that occur during initial learning in children. Therefore, it is critical to study in what respect brain activity changes *during* arithmetic learning in children.

[1] In a recent study, Bloechle et al. (2016) also speculated that the activation of the left temporoparietal cortex (particularly at the level of the AG) during arithmetic tasks might be explained by the involvement of this region in attention to memory (Cabeza, Ciaramelli, & Moscovitch, 2012). That is, solutions of well-known problems encoded in long-term memory may enter working memory during arithmetic tasks and capture bottom-up attention. Although this proposal differs from the DMN account because it posits that left temporoparietal activity specifically reflects a switch in attentional demands, both accounts are similar in that they assume that this region may not be involved in fact retrieval per se.

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Evidence From Studies in Children

Several longitudinal or cross-sectional neuroimaging studies of arithmetic learning in children have been performed over the past few years. Somewhat surprisingly, these studies have provided very little support for age-related (or fluency-related) increases of activity in the left temporoparietal cortex. In a seminal cross-sectional study, Rivera, Reiss, Eckert, and Menon (2005) used a single-digit addition and subtraction task to investigate the neural changes associated with arithmetic learning in children and adults. They found age-related increases of activity in the left anterior IPS but not in the left AG or around the regions of the STG/MTG, which are typically associated with verbal-phonological processing. Rosenberg-Lee, Barth, and Menon (2011) further found greater activity in third than second graders for single-digit addition in the right but not left AG and no change of activity in either the left STG or the left MTG. Kucian, von Aster, Loenneker, Dietrich, and Martin (2008) found greater activity in adults than children in the left IPS but not in the left temporoparietal cortex in a single-digit addition task. In yet another study using a single-digit addition task in children, Cho et al. (2012) found that the only region in which there was an age-related increase of activity was the right superior parietal cortex. Finally, Qin et al. (2015) found longitudinal increases of activity in prefrontal, posterior parietal, and occipital cortex in an addition task but again not in left temporoparietal cortex.

How can one make sense of the discrepancy between adult studies showing activation of left temporoparietal regions during arithmetic tasks on the one hand and developmental studies showing a lack of age-related changes of activity in these regions on the other hand? There might be at least two ways to explain this discrepancy. First, it is clear from the review of adult neuroimaging studies that not all arithmetic tasks recruit left temporoparietal mechanisms. For example, these regions appear to be more activated in single-digit multiplication tasks than in single-digit addition and subtraction tasks (Lee, 2000; Prado et al., 2011; Zhou et al., 2007). This is also consistent with a study by Delazer et al. (2005), showing that only facts that are learned by drill might lead to increased activity in the left AG. It is thus interesting to note that all of the developmental studies described earlier have used tasks that involve arithmetic skills that are (largely) never drilled in school, such as single-digit addition and subtraction problems. Rather, such problems are mastered in adults because they are repeatedly practiced over the years. Therefore, it remains possible that age-related increases of activity would be observed in the left temporoparietal cortex for problems that are drilled, i.e., single-digit multiplication problems. Second, it is also possible that studies might have missed developmental changes of activity in the left temporoparietal cortex because these changes are

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subtle and require substantial power to be detected with whole-brain analyses of brain imaging data (with larger sample sizes needed to detect relatively small effects).

Prado et al. (2014) attempted to shed light on these two assumptions by investigating the neural bases of single-digit multiplication problem solving in a cross-sectional study of 34 children from second to seventh grade. Critically, brain activity was measured in the whole brain and in a specific region of the left MTG that was localized using a word-rhyming task. Multiplication problems were found to be associated with a developmental increase of activity in that specific MTG cluster. This increase of activity, however, was absent in a single-digit subtraction task. Therefore, at least as far as single-digit multiplication problems are concerned, greater involvement of mechanisms supporting verbal-phonological processing can be observed. This is not to say that learning other types of arithmetic facts cannot engage left temporoparietal regions in children. For example, recent studies in 9- to 12-year-olds have found greater activity in the left AG for (1) small versus large addition and subtraction (De Smedt, Holloway, & Ansari, 2011) and (2) symbolic versus nonsymbolic subtraction across participants (Peters, Polspoel, Op de Beeck, & De Smedt, 2016). Therefore, learning addition facts might also rely to some extent on verbal-phonological mechanisms, albeit perhaps less uniformly than multiplication facts.

Overall, the evidence reviewed earlier suggests that integrity of the phonological processing mechanisms supporting the acquisition of reading might influence at least some aspects of the neural mechanisms that support arithmetic learning. This hypothesis has been recently tested in a study by Evans, Flowers, Napoliello, Olulade, and Eden (2014). The authors investigated the neural bases of arithmetic processing in 14 children with developmental dyslexia (compared with 14 typically developing children). Previous neuroimaging studies have found that individuals with dyslexia have anatomic and functional impairments in brain regions supporting phonological processing in the left temporoparietal cortex (Richlan, Kronbichler, & Wimmer, 2009). Behavioral studies have also identified arithmetic difficulties in children with dyslexia (Simmons & Singleton, 2008). Therefore, it is possible that these difficulties lead to abnormal processing of arithmetic facts in left temporoparietal regions. In line with this hypothesis, less activity was found in the left supramarginal gyrus, a region adjacent to both the left STG and the left AG, in response to single-digit addition and subtraction in dyslexic compared with typically developing children. Thus, it is possible that anatomic and functional impairments in left temporoparietal phonological mechanisms might affect at least some aspects of arithmetic processing in dyslexic children. However, as we have seen here, the involvement of temporoparietal mechanisms in arithmetic learning is very task specific. It is thus unlikely to account in itself for the strong

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relationship between arithmetic learning and reading acquisition in children. Other explanations might then need to be considered. In the following, we discuss the recent proposal that learning arithmetic and learning to read may both require the skill to automatize procedures until complete fluency.

PROCEDURAL AUTOMATIZATION: A COMMON DENOMINATOR BETWEEN LEARNING TO READ AND LEARNING ARITHMETIC?

Perhaps the most striking and intuitive similarity between learning to read and learning arithmetic is that they arguably share a similar goal: both involve automatizing a skill until complete fluency. By the end of elementary school, children are expected to read and understand common words with no apparent effort in much the same way as they should quickly respond 5 when faced with 2+3. Interestingly, it has long been proposed that a domain-general deficit in procedural automatization may be at the source of dyslexia in children (Lum, Ullman, & Conti-Ramsden, 2013; Nicolson & Fawcett, 2007), and this hypothesis has been recently extended to dyscalculia (Evans & Ullman, 2016). This raises the possibility that at least some aspects of arithmetic learning and reading acquisition both rely on the memory system that supports the automatization of procedures through practice, i.e., procedural memory. This hypothesis is interesting for the current purpose of this chapter because it may explain some of the interactions between learning to read and learning arithmetic. In the following, we describe how theories have explained dyslexia in terms of impaired automatization of procedures and how these theories have been recently extended to dyscalculia. We then examine to what extent this idea is supported by developmental behavioral and neuroimaging research.

The Procedural Learning View of Dyslexia and Dyscalculia

Clearly, the most popular explanation of dyslexia is that it is caused by a deficit in accessing and manipulating phonological information (Vellutino et al., 2004). This phonological deficit view is widely supported by studies showing that children with dyslexia do have impairments in phonological processing, including phonological awareness (sensitivity to the sound structure of oral language) (Vellutino et al., 2004). Yet, studies also show that children with dyslexia often exhibit impairments in other domains, such as attention (Facoetti, Paganoni, Turatto, Marzola, & Mascetti, 2000; Varvara, Varuzza, Sorrentino, Vicari, & Menghini, 2014), motor control (Fawcett & Nicolson, 1995;

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Nicolson & Fawcett, 1994), and implicit sequence learning (Gabay, Thiessen, & Holt, 2015; Hedenius et al., 2013; Kelly, Griffiths, & Frith, 2002). Because these impairments are not easily explained by a phonological deficit account, several researchers have argued that dyslexia may also arise from a more general learning disorder (Lum et al., 2013; Nicolson & Fawcett, 2007; Nicolson, Fawcett, & Dean, 2001; Ullman, 2004). Specifically, it has been proposed that dyslexia may stem from a general impairment in the learning and memory system that supports the acquisition of skills and habits through repeated practice, i.e., procedural memory (Ullman, 2016, pp. 953–968). Not only can this hypothesis explain the range of nonreading deficits observed in dyslexic children but also it can account for the reading deficits. That is, impairments in the procedural memory system may disrupt "automatization of skill and knowledge, which may potentially affect grapheme–phoneme conversion, word recognition, verbal working memory, and learning orthographic regularities, thereby contributing to reading impairment" (Gabay et al., 2015, p. 935). The brain system supporting procedural memory has been documented in the literature. It relies on a network of brain regions, which includes the basal ganglia, the premotor cortex, the cerebellum, and parts of the inferior parietal cortex (Ullman, 2016, pp. 953–968). Interestingly, brain imaging studies have found that children with dyslexia do exhibit anatomic and functional impairments in several of these regions, including the basal ganglia (Brunswick, McCrory, Price, Frith, & Frith, 1999; Kita et al., 2013), cerebellum (Eckert et al., 2003; Pernet, Poline, Demonet, & Rousselet, 2009; Rae et al., 2002), frontal cortex (Eckert et al., 2003; Richlan et al., 2009), and parietal cortex (Richlan et al., 2009).

Recently, Evans and Ullman (2016) have proposed an extension of the procedural learning deficit theory to dyscalculia (Evans & Ullman, 2016). This extension is partly motivated by the observed comorbidity between dyslexia and dyscalculia, estimated at around 40% (Wilson et al., 2015). Evans and Ullman (2016) proposed that this comorbidity may be explained by the fact that (at least) some children may have a general learning deficit that would affect both reading and arithmetic learning. This deficit may stem from impaired procedural memory. The proposal relies on the assumption that, much like learning to read, acquiring arithmetic skills involves using increasingly efficient procedures that need to be automatized by the end of the learning process. Impairments in procedural memory may hinder this progressive automatization of procedures, leading to both dyslexia and dyscalculia. Although that specific hypothesis remains to be thoroughly explored, we will see in the following that there is growing support for the idea that automatized procedures may indeed underlie at least some aspects of arithmetic skills in typically developing children and educated adults.

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Arithmetic Learning and the Use of Increasingly Efficient Procedures

A defining feature of arithmetic acquisition in children is the use of increasingly efficient procedures over the course of learning. This has been consistently demonstrated in studies in which children are asked to report the strategies they use when solving problems (Baroody & Tiilikainen, 2003; Carpenter & Moser, 1984). Consider, for example, the strategies that young children report relying on in simple addition tasks. Using external aids such as fingers or objects, young children may first use what is often called the *counting-all* procedure: they count out two sets of objects before combining them and counting the newly formed set. Older children may then realize that objects can be replaced by counting words. This realization is often accompanied by the appearance of the more sophisticated *counting-on* procedure, according to which children start to count from one of the two number words. In using this *counting-on* procedure, many children will also realize that starting from the largest number (*minimum strategy*) is more efficient than starting from the smaller number (*maximum strategy*).

In many ways, this use of increasingly efficient procedures over development parallels what is observed when children learn to read. That is, becoming an efficient reader also involves a refinement of procedures underlying word decoding (Farrington-Flint, Coyne, Stiller, & Heath, 2008). For example, young children typically start by identifying words using a *sounding out* strategy, in which graphemes are matched onto phonemes. This *sounding out* strategy can be far from accurate because in many languages (but not all) words may have irregular mappings from orthography to phonology (Farrington-Flint et al., 2008). With reading practice, the *sounding out* strategy will be gradually replaced by more sophisticated (and more efficient) procedures. For instance, a more elaborate procedure might involve making an analogy from the spelling sound pattern of a familiar word to an unfamiliar one. An even more efficient procedure might involve using morphological rules (Farrington-Flint et al., 2008). Therefore, both learning to read and learning arithmetic are characterized by the use of increasingly efficient procedures with age.

Procedural Automatization as a Critical Element of Arithmetic Fluency

Of course, the end product of both learning to read and learning arithmetic ought to be fluency. For instance, educated adults must be able to very quickly recognize a written word. They must also be able to quickly come up with the answer of a single-digit addition problem. Over the past decades, the dominant view has been that procedures

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can never be efficient enough to attain arithmetic fluency and that there is necessarily a shift from procedural to retrieval-based strategies over development (Barrouillet & Fayol, 1998; Groen & Parkman, 1972). That is, the repetitive use of counting procedures over the course learning is thought to lead to the repetitive co-occurrence of operands and answers in working memory, which would result in the emergence of associations between numbers (Logan, 1988). Eventually, adults and educated children might then directly retrieve these associations from declarative memory when faced with simple arithmetic problems—without relying on procedural knowledge (Siegler & Shipley, 1995, pp. 31–76).

More recently, however, several studies have suggested that counting procedures might be so practiced over the course of learning that they might actually be executed automatically and unconsciously in adults and educated children. These automatized procedures may then be very efficient and also account for basic arithmetic skills of fluent individuals. Evidence for this alternative view comes from behavioral studies showing that adults systematically show a problem size effect when they solve even very simple addition problems (i.e., problems with operands smaller than 4) (Barrouillet & Thevenot, 2013; Thevenot, Barrouillet, Castel, & Uittenhove, 2015; Uittenhove, Thevenot, & Barrouillet, 2015). Specifically, the time it takes to solve these problems increases linearly with the distance between the original operand and the sum (i.e., adults take 20 ms longer to solve 1 + 3 than 1 + 2 and 20 ms longer to solve 1 + 4 than 1 + 3) (Barrouillet & Thevenot, 2013; Uittenhove et al., 2015) (see Fig. 2.2). This pattern is difficult to explain with the view that associations are retrieved from a network of facts. Rather, it suggests that, without being aware of it, adults might solve these problems by using an automatized counting procedure that may involve scanning a sequence of numbers oriented from left to right (i.e., the so-called mental number line or MNL) (Barrouillet & Thevenot, 2013; Mathieu, Gourjon, Couderc, Thevenot, & Prado, 2016). Solving time would then depend on the distance between the original operand and the target sum to be reached. In support for this view, Mathieu et al. (2016) have recently found that solving addition and subtraction problems is associated with rightward and leftward shifts of attention (respectively) in adults. Such an automatized procedure may be the result of extensive practice with counting in children, in line with the long-standing idea that the repetitive practice of a procedure can lead to its automatization (Baroody, 1983). Therefore, without denying that retrieval from memory might occur in some instances, it is conceivable that learning arithmetic might also rely on the progressive automatization of procedures and therefore involve to a significant extent procedural memory.

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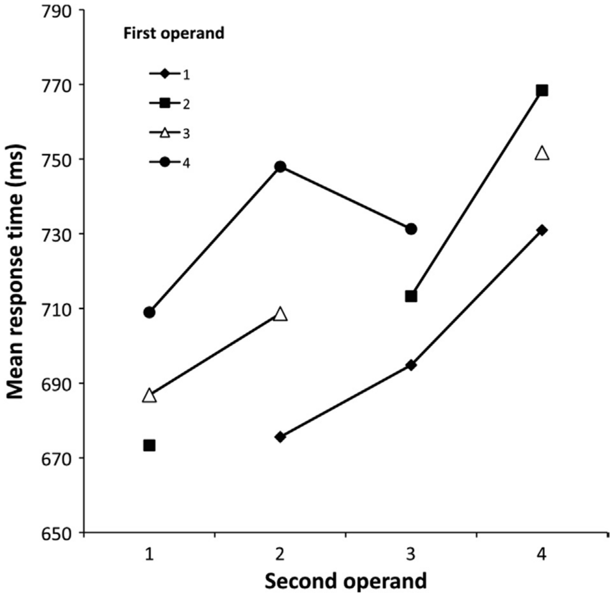

FIGURE 2.2 Mean resolution times of simple addition problems (with operands from 1 to 4 but not including tie problems) as a function of the magnitude of the first and second operands in adults. *Reproduced from Barrouillet, P., & Thevenot, C. (2013). On the problem-size effect in small additions: Can we really discard any counting-based account?* Cognition, 128*(1), 35–44.*

Developmental Neuroimaging Evidence for Procedural Automatization

The idea that arithmetic learning and reading acquisition gradually involves an automatization of procedural knowledge makes at least three interesting neural predictions. First, if procedures become automatic with expertise, they should require fewer and fewer executive resources. This should translate into decreases of activity in brain regions that support executive control and working memory. This idea is fairly well supported by developmental neuroimaging studies of arithmetic learning. Indeed, studies have consistently demonstrated decreases of activity in regions of the frontal cortex with age in arithmetic tasks. For example, this developmental pattern was first observed in the seminal cross-sectional study by Rivera et al. (2005). There, the development of skills for solving addition problems from ages 8 to 19 was related to decreases of activity in several regions of the inferior, middle, and superior frontal gyri. Rosenberg-Lee, Barth, et al. (2011) similarly found less activity in the ventral medial prefrontal cortex in third graders compared with second graders in an addition task. Recently, longitudinal decreases of activity were found in the bilateral dorsolateral prefrontal cortex as children in elementary school become increasingly proficient in solving addition problems (Qin et al., 2015). Decreases of activity in prefrontal cortex were also noticed in subtraction and multiplication problem solving in a cross-sectional study of children from second to seventh grade (Prado et al., 2014). Finally, differences in prefrontal activity are also observed between adults and children in arithmetic tasks (with less activity in adults than children) (Kucian et al., 2008).

Second, if children increasingly rely on procedures that involve automatized shifts of attention along the MNL (Barrouillet & Thevenot, 2013; Mathieu et al., 2016; Thevenot et al., 2015; Uittenhove et al., 2015), one should observe developmental increases of arithmetic-related activity in the posterior superior parietal lobule, the main region that is thought to support attentional orientation along the MNL (Dehaene et al., 2003). This is exactly what we observed in a recent cross-sectional study in which fMRI activity of children from second to seventh grade was measured while they were solving single-digit problems (Prado et al., 2014). Importantly, these changes were operation specific as they were observed for subtraction problems (a type of operation that is not learned by rote in school) but not for multiplication problems (which are mostly learned by rote). More generally, these results are consistent with studies showing that age-related decreases of prefrontal activity in arithmetic tasks are often accompanied by age-related increases of activity in regions of the parietal cortex (Prado et al., 2014; Rivera et al., 2005; Rosenberg-Lee, Barth, et al., 2011). This has led several researchers to propose that arithmetic learning may be characterized by a frontal-to-parietal developmental shift of activity, with parietal regions becoming progressively specialized for arithmetic tasks (Ansari, 2008). This increase in specialization of the parietal cortex for arithmetic tasks is broadly consistent with the claim that parietal regions might increasingly support automatized numerical-based procedures with development (Prado et al., 2014).

Interestingly, a similar shift from prefrontal to posterior brain regions can be observed during reading acquisition. For example, Martin et al. (2015) recently compared in a metaanalysis the results of 20 adult neuroimaging studies on reading with the results of 20 neuroimaging studies performed with children. They only found two brain regions that were more consistently activated across children studies than across adult studies: the left STG and the medial prefrontal cortex. This is consistent with the idea that, compared with adults, children might use phonology-based strategies that might be more effortful and require executive control mechanisms in the prefrontal cortex (as well as access to phonological representation in the STG). In contrast, compared with children studies, adult studies more consistently activated posterior regions, such as the ventral occipital cortex and the cerebellum. This latter finding points to an anterior-to-posterior shift of activity for reading with development, echoing the frontal-to-parietal shift observed during arithmetic learning.

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A third hypothesis, related to the idea that both arithmetic learning and reading acquisition involve an automatization of procedural knowledge, is that arithmetic and reading disabilities should be related to impairments in brain structures that underlie procedural memory. These may include the posterior parietal cortex, the cerebellum, and subcortical structures such as the basal ganglia (Evans & Ullman, 2016). All of these regions are known to be affected in dyslexia (Richlan et al., 2009). Research on dyscalculia also suggests impairments in regions that support procedural memory. Arguably the most consistent locus of anatomic and functional impairments in dyscalculia is the posterior parietal cortex (Ashkenazi, Rosenberg-Lee, Tenison, & Menon, 2012; Iuculano et al., 2015; Molko et al., 2003; Rosenberg-Lee et al., 2015; Rotzer et al., 2008, 2009; Rykhlevskaia, Uddin, Kondos, & Menon, 2009). However, abnormalities have also been observed in the basal ganglia (Molko et al., 2003) and the cerebellum (Rykhlevskaia et al., 2009). Therefore, although more studies are needed, the available evidence suggests that at least some structures supporting procedural memory may be impaired in dyscalculia.

CONCLUSION AND FUTURE DIRECTIONS

Cognitive neuroscience studies largely indicate that human mathematical skills are rooted in nonverbal mechanisms. It is thus somewhat paradoxical that there appears to be a link between the development of arithmetic and reading skills in children (Durand et al., 2005; Hart et al., 2009; Hecht et al., 2001; Wilson et al., 2015). The goal of this chapter was to provide an overview of two possible explanations for this link and confront these explanations to available evidence from developmental cognitive neuroscience.

First, the triple-code model suggests that answers of the most familiar arithmetic facts may be increasingly retrieved from verbal-phonological codes as individuals become fluent (Dehaene et al., 2003). Yet, developmental cognitive neuroscience studies provide limited evidence for this assumption (Cho et al., 2012; Rivera et al., 2005; Rosenberg-Lee, Chang, Young, Wu, & Menon, 2011). One study suggests that increases of activity may be observed in the left temporal cortex as children become increasingly proficient with single-digit multiplication problems (Prado et al., 2014), but this effect appears to be task dependent. The contribution of verbal-phonological mechanisms to arithmetic learning may thus be restricted to facts that are explicitly learned by rote in school.

Second, it has been proposed that learning arithmetic and learning to read may both rely on the automatization of rules and procedures (Barrouillet & Thevenot, 2013; Thevenot et al., 2015; Uittenhove et al., 2015). Procedural memory systems may thus be critical to the acquisition

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of both skills, and impairments in procedural memory may be at the source of both dyscalculia and dyslexia. Not only does that hypothesis explain why one of the most consistent results obtained in developmental neuroimaging studies is an increase of activity in the parietal cortex (rather than in the left temporoparietal cortex) but also it highlights the importance of procedural memory for arithmetic learning. Because procedural memory has long been hypothesized to also be central to reading acquisition (Ullman, 2016, pp. 953–968), this idea may explain the link between arithmetic and reading acquisition and the comorbidity between dyslexia and dyscalculia (Wilson et al., 2015).

Overall, given the limited neurodevelopmental support for the idea that verbal-phonological processing underlies arithmetic learning in children, the procedural hypothesis is an interesting explanation for the link between arithmetic and reading skills. Of course, this does not mean that other domain-general factors cannot also account for that link in some children. For example, it is clear that both arithmetic and reading tasks involve working memory, attention, or cognitive control (Ashkenazi et al., 2013). It is possible that disruptions in these domain-general mechanisms may also lead to both reading and arithmetic impairments in children (as well as impairments in other skills). This is generally consistent with the idea that arithmetic learning involves a wide range of skills and that dyscalculia may be a heterogeneous disorder (Fias, Menon, & Szucs, 2013). Nevertheless, learning to read and learning arithmetic may both place important demands on procedural memory and automatization of skills, a factor that may explain a large part of the relationship between arithmetic and reading performance in children.

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